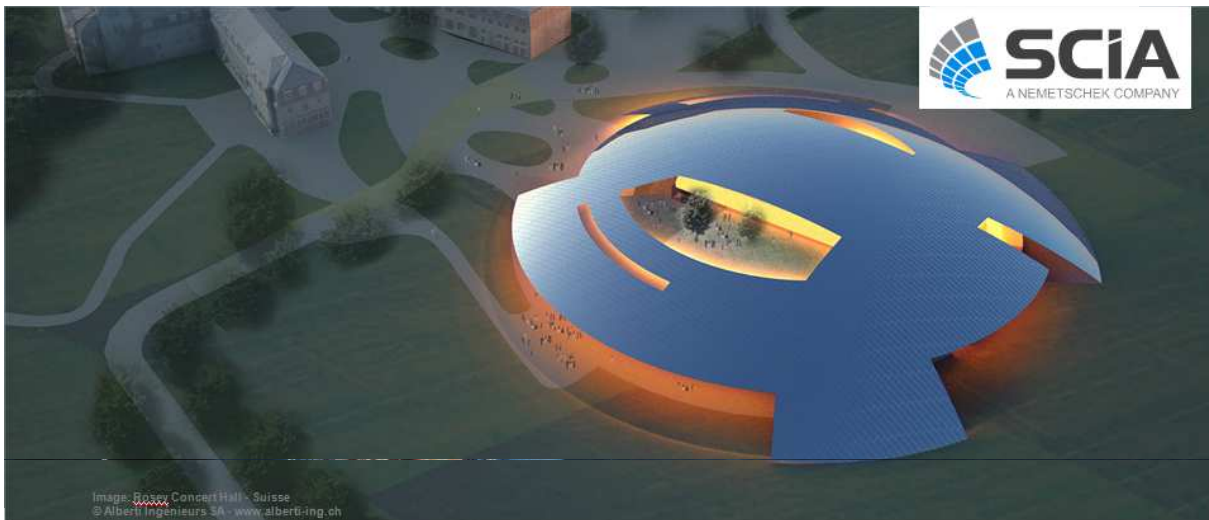


## Eurocode Training

### EN 1992-1-1: Reinforced Concrete

All information in this document is subject to modification without prior notice. No part of this manual may be reproduced, stored in a database or retrieval system or published, in any form or in any way, electronically, mechanically, by print, photo print, microfilm or any other means without prior written permission from the publisher. SCIA is not responsible for any direct or indirect damage because of imperfections in the documentation and/or the software.

© Copyright 2015 SCIA nv. All rights reserved.



## Eurocode Training

### EN 1992-1-1: Reinforced Concrete

**SCIENGINEER**

## Introduction



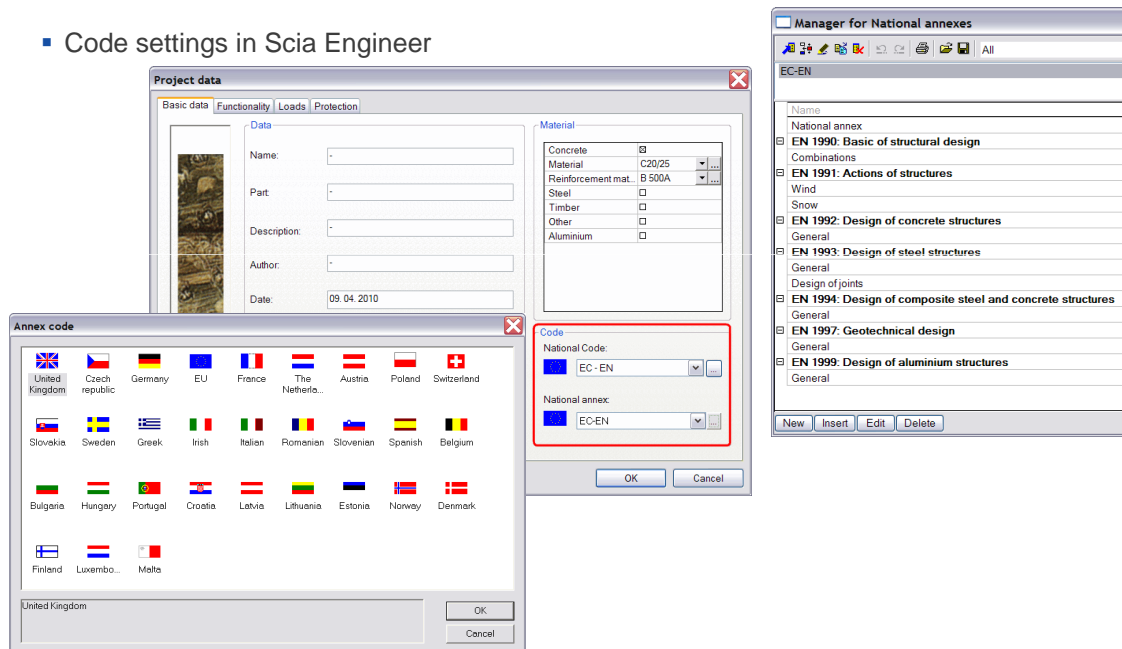
Subject of this workshop = the European Standard EN 1992

### **Eurocode 2: Design of concrete structures**

#### **Part 1-1: General rules and rules for buildings**

which has been prepared by Technical Committee CEN/TC250 «Structural Eurocodes»

## Code settings in Scia Engineer



2

## Section 1 General

3

- Design of buildings and civil engineering works in plain, reinforced and prestressed concrete
- Requirements for resistance, serviceability, durability and fire resistance of concrete structures
- Eurocode 2 is subdivided into the following parts:
  - Part 1-1: General rules and rules for buildings
  - Part 1-2: Structural fire design
  - Part 2: Reinforced and prestressed concrete bridges
  - Part 3: Liquid retaining and containing structure

- **Focus in this workshop: Reinforced concrete – Part 1-1**

- General basis for design of structures in plain, reinforced and prestressed concrete made with normal and light weight aggregates together with specific rules for buildings
  - Section 1: General
  - Section 2: Basis of design
  - Section 3: Materials
  - Section 4: Durability and cover to reinforcement
  - Section 5: Structural analysis
  - Section 6: Ultimate limit states
  - Section 7: Serviceability limit states
  - Section 8: Detailing of reinforcement and prestressing tendons - General
  - Section 9: Detailing of members and particular rules
  - Section 10: Additional rules for precast concrete elements and structures
  - Section 11: Lightweight aggregate concrete structures
  - Section 12: Plain and lightly reinforced concrete structures

- **Focus in this workshop: Sections 1 to 9**

## Section 2 Basis of design

## Section 2 Basis of design

### Material and product properties

- general rules → see EN 1990 Section 4
- specific provisions for concrete & reinforcement → see EN 1992 Section 3

### Shrinkage and creep

- time dependent
- effects have to be taken into account in the SLS
- in the ULS: only if significant (for example 2<sup>nd</sup> order)
- quasi-permanent combination of loads

Application in Scia Engineer:

Creep: CDD or PNL calculation

Shrinkage: TDA calculation

### Design values

#### Partial factors for shrinkage, prestress, fatigue loads

#### Partial factors for materials

- ULS: recommended values

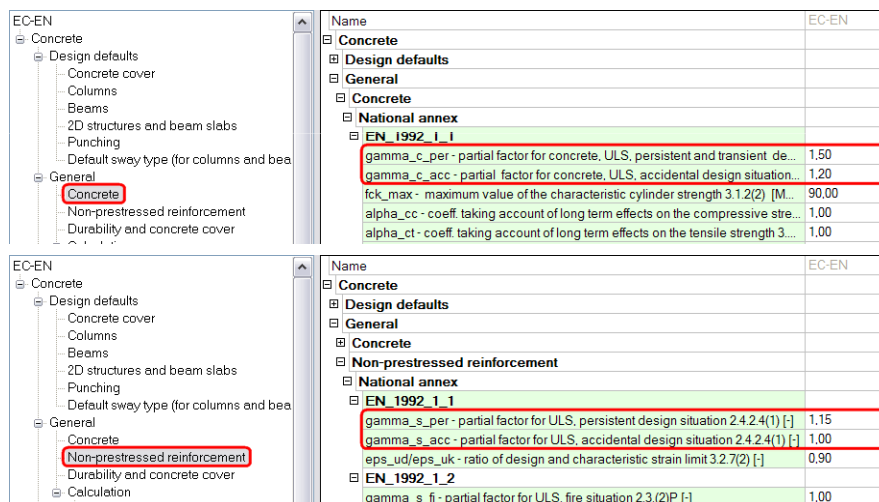
Design situations	$\gamma_c$ for concrete	$\gamma_s$ for reinforcing steel	$\gamma_s$ for prestressing steel
Persistent & Transient	1,5	1,15	1,15
Accidental	1,2	1,0	1,0

- SLS: recommended values

$$\gamma_c \text{ and } \gamma_s = 1$$

### Design values

#### Partial factors in Scia Engineer



The screenshot displays the 'Design defaults' section in Scia Engineer, showing the 'Concrete' and 'Non-prestressed reinforcement' settings. The 'Concrete' settings are highlighted with a red box, and the 'Non-prestressed reinforcement' settings are also highlighted with a red box.

Name	EC-EN
Concrete	EC-EN
Design defaults	
General	
Concrete	
National annex	
EN_1992_1_1	
gamma_c_per - partial factor for concrete, ULS, persistent and transient de...	1.50
gamma_c_acc - partial factor for concrete, ULS, accidental design situation...	1.20
fck_max - maximum value of the characteristic cylinder strength 3.1.2(2) [M...	90.00
alpha_cc - coeff. taking account of long term effects on the compressive stre...	1.00
alpha_ct - coeff. taking account of long term effects on the tensile strength 3...	1.00

Name	EC-EN
Concrete	EC-EN
Design defaults	
General	
Concrete	
Non-prestressed reinforcement	
National annex	
EN_1992_1_1	
gamma_s_per - partial factor for ULS, persistent design situation 2.4.2.4(1) [-]	1.15
gamma_s_acc - partial factor for ULS, accidental design situation 2.4.2.4(1) [-]	1.00
eps_ud/eps_uk - ratio of design and characteristic strain limit 3.2.7(2) [-]	0.90
EN_1992_1_2	
gamma_s_fi - partial factor for ULS, fire situation 2.3.(2)P [-]	1.00

## Section 3 Materials

10

## Section 3 Materials

### Characteristic strength

#### Compressive strength

- denoted by concrete strength classes (e.g. C25/30)
- cylinder strength  $f_{ck}$  and cube strength  $f_{ck,cube}$
- $f_{ck}$  determined at 28 days

If required to specify  $f_{ck}(t)$  at time  $t$  for a number of stages  
(e.g. demoulding, transfer of prestress):

- $f_{ck}(t) = f_{cm}(t) - 8 \text{ [MPa]}$  for  $3 < t < 28$  days
- $f_{ck}(t) = f_{ck}$  for  $t \geq 28$  days

where  $f_{cm}(t) = \beta_{cc}(t) f_{cm}$

with  $\beta_{cc}(t)$  dependent on the cement class

11



EN Table 3.1 Strength and deformation characteristics for concrete

Strength classes for concrete														Analytical relation / Explanation
$f_{ck}$ (MPa)	12	16	20	25	30	35	40	45	50	55	60	70	80	90
$f_{d,cube}$ (MPa)	15	20	25	30	37	45	50	55	60	67	75	85	95	105
$f_{cm}$ (MPa)	20	24	28	33	38	43	48	53	58	63	68	78	88	98
$f_{cm}$ (MPa)	1,6	1,9	2,2	2,6	2,9	3,2	3,5	3,8	4,1	4,2	4,4	4,6	4,8	5,0
$f_{ak,0.95}$ (MPa)	1,1	1,3	1,5	1,8	2,0	2,2	2,5	2,7	2,9	3,0	3,1	3,2	3,4	3,5
$f_{ak,0.95}$ (MPa)	2,0	2,5	2,9	3,3	3,8	4,2	4,6	4,9	5,3	5,5	5,7	6,0	6,3	6,6
$E_{cm}$ (GPa)	27	29	30	31	33	34	35	36	37	38	39	41	42	44
$\epsilon_{c1}$ (‰)	1,8	1,9	2,0	2,1	2,2	2,25	2,3	2,4	2,45	2,5	2,6	2,7	2,8	2,8
$\epsilon_{c2}$ (‰)	3,5									3,2	3,0	2,8	2,8	2,8
$\epsilon_{c2}$ (‰)	2,0									2,2	2,3	2,4	2,5	2,6
$\epsilon_{c2}$ (‰)	3,5									3,1	2,9	2,7	2,6	2,6
$n$	2,0									1,75	1,6	1,45	1,4	1,4
$\epsilon_{c3}$ (‰)	1,75									1,8	1,9	2,0	2,2	2,3
$\epsilon_{c3}$ (‰)	3,5									3,1	2,9	2,7	2,6	2,6

Material characteristics in Scia Engineer

Materials

C12/15

C16/20

C20/25

C25/30

C30/37

C35/45

C40/50

C45/55

C50/60

C55/67

C60/75

C70/85

C80/95

C90/105

C55/67(EN1992-2)

C60/75(EN1992-2)

C70/85(EN1992-2)

C80/95(EN1992-2)

C90/105(EN1992-2)

G modulus [MPa]

Log decrement

Colour

Specific heat [J/gK]

Temperature dependency of specific heat

Thermal conductivity [W/mK]

Temperature dependency of thermal conductivity

Order in code

EN 1992-1-1

Characteristic compressive cylinder strength  $f_{ck}(28)$  [MPa]

Calculated depended values

Mean compressive strength  $f_{cm}(28)$  [MPa]

$f_{cm}(28) - f_{ck}(28)$  [MPa]

Mean tensile strength  $f_{ctm}(28)$  [MPa]

$f_{ctk} 0.05(28)$  [MPa]

$f_{ctk} 0.95(28)$  [MPa]

Design compressive strength - persistent ( $f_{cd} = f_{ck} / \gamma_{c,p}$ ) [MPa]

Design compressive strength - accidental ( $f_{cd} = f_{ck} / \gamma_{c,a}$ ) [MPa]

Strain at reaching maximum strength  $\epsilon_{c2}$  [1e-4]

Ultimate strain  $\epsilon_{cu2}$  [1e-4]

Strain at reaching maximum strength  $\epsilon_{c3}$  [1e-4]

Ultimate strain  $\epsilon_{cu3}$  [1e-4]

Stone diameter (dg) [mm]

Cement class

Type of aggregate

Measured values

Stress-strain diagram

Type of diagram

Picture of Stress-strain diagram

13667e+04

02

6.0000e-01

None

4.5000e-01

None

5

30.00

0

38.00

8.00

2.90

2.00

3.80

20.00

25.00

20.0

35.0

17.5

35.0

32

N (normal hardening - CEM 32.5 R, CEM 42.5 N)

Quartzite

Measured values of mean compressive strength (influence of ageing)

Stress-strain diagram

Bi-linear stress-strain diagram

### Design strength

#### Design compressive strength

$$f_{cd} = \alpha_{cc} f_{ck} / \gamma_C \quad (3.15)$$

#### Design tensile strength

$$f_{ctd} = \alpha_{ct} f_{ctk,0,05} / \gamma_C \quad (3.16)$$

$\alpha_{cc}$  (resp.  $\alpha_{ct}$ ) is a coefficient taking account of **long term effects** on the compressive strength (resp. tensile strength) and of unfavourable effects resulting from the way the load is applied.

$$\alpha_{cc} = 1,0 \text{ and } \alpha_{ct} = 1,0 \quad (\text{recommended values})$$

### Elastic deformation

- dependent on composition of the concrete (especially the aggregates)
- approximate values for modulus of elasticity  $E_{cm}$ , secant value between  $\sigma_c = 0$  and  $0,4f_{cm}$ : see EN Table 3.1(for quartzite aggregates)
  - Reduction for limestone aggregates (10%) – sandstone aggregates (30%)
  - Augmentation for basalt aggregates (20%)

- variation of the modulus of elasticity with time:

$$E_{cm}(t) = (f_{cm}(t) / f_{cm})^{0,3} E_{cm} \quad (3.5)$$

- Poisson's ratio: 0,2 for uncracked concrete and 0 for cracked concrete

### Creep and shrinkage

dependent on

- ambient humidity
- dimensions of the element
- composition of the concrete
- maturity of the concrete at first loading
- duration and magnitude of the loading

### Creep coefficient in Scia Engineer

- SLS
  - General
  - Creep**
  - Crack proof
  - Code Dependent Deflections
- Detailing provisions
  - Columns
  - Beams
  - 2D structures and slabs

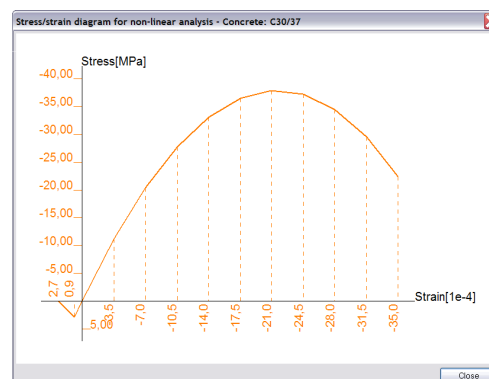
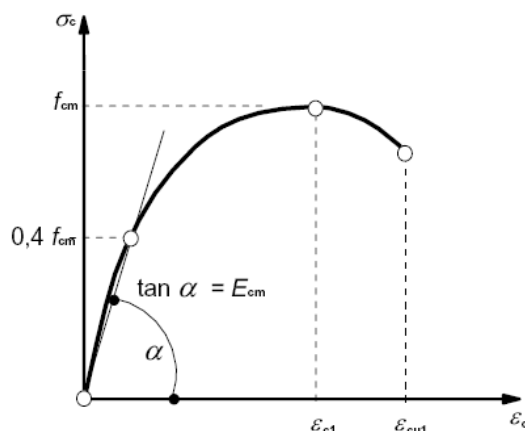
Creep	
Creep for concrete - Code dependent deflections (CDD)	
Creep coefficient [-]	2.50
Calculate creep coefficient	<input checked="" type="checkbox"/> yes
Relative humidity [%] [-]	50.00
Age at loading [day]	28
Age at concrete [day]	1825

## Section 3 Materials

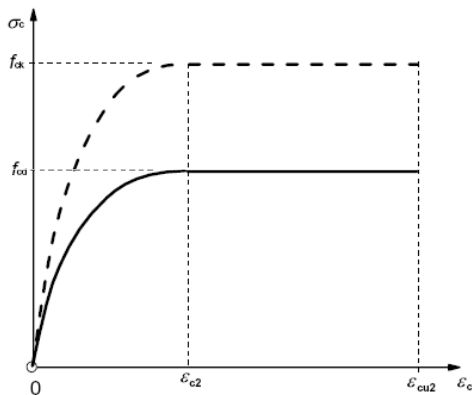
### Stress-strain relation for non-linear structural analysis

$$\frac{\sigma_c}{f_{cm}} = \frac{k\eta - \eta^2}{1 + (k-2)\eta}$$

Remark: the use of  $0,4f_{cm}$  for the definition of  $E_{cm}$  is approximate!

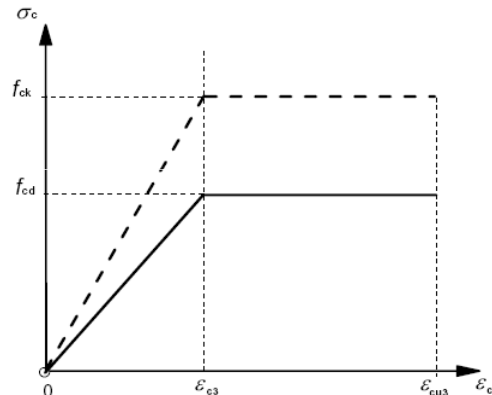


### Stress-strain relations for the design of cross-sections



$\epsilon_{c2,3}$  : strain at reaching the maximum strength

$\epsilon_{cu2,3}$  : ultimate strain

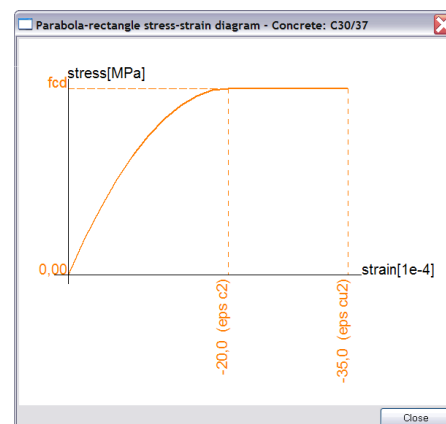
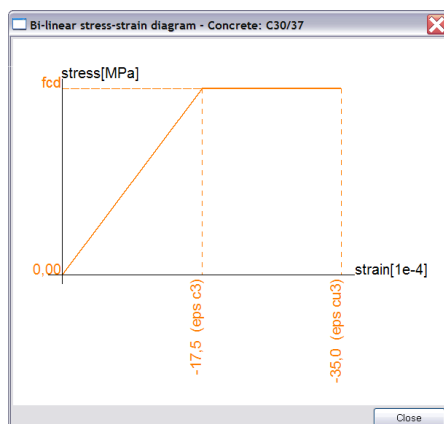


(see EN Table 3.1)

## Section 3 Materials

### Stress-strain relations for the design of cross-sections in Scia Engineer

Stress-strain diagram	
Type of diagram	Bi-linear stress-strain diagram
Picture of Stress-strain diagram	Bi-linear stress-strain diagram
	Parabola-rectangle stress-strain diagram

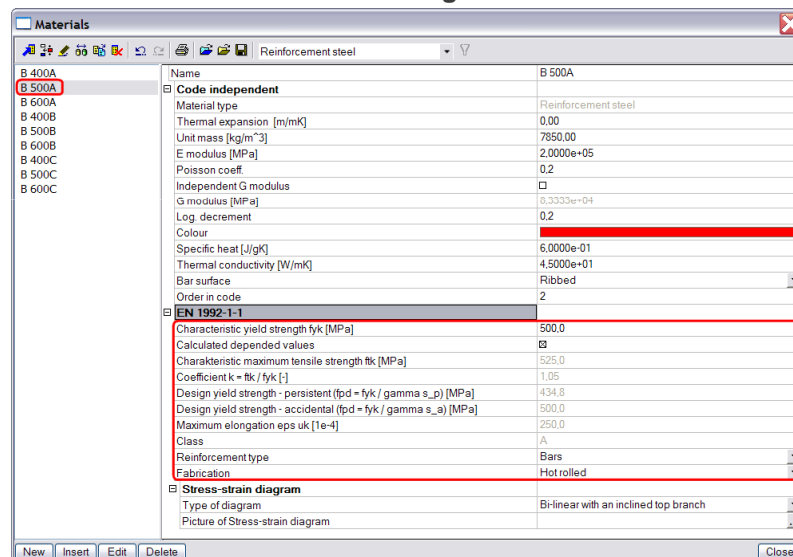


### Properties

The behaviour of reinforcing steel is specified by the following properties:

- yield strength ( $f_{yk}$  or  $f_{0,2k}$ )
- maximum actual yield strength ( $f_{y,max}$ )
- tensile strength ( $f_t$ )
- ductility ( $\epsilon_{uk}$  and  $f_t/f_{yk}$ )
- bendability
- bond characteristics ( $f_R$ : see Annex C)
- section sizes and tolerances
- fatigue strength
- weldability
- shear and weld strength for welded fabric and lattice girders

### Material characteristics in Scia Engineer



The screenshot shows the 'Materials' dialog box in Scia Engineer. The 'Reinforcement steel' category is selected. The material 'B 500A' is highlighted in the list on the left. The properties are displayed in a table:

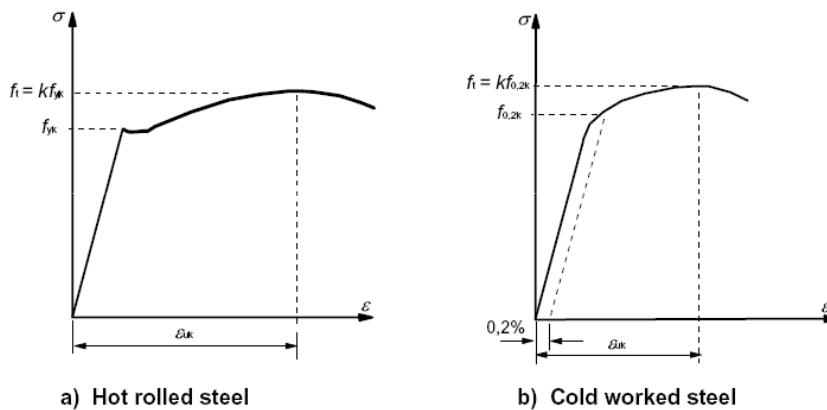
Name	B 500A
<b>Code independent</b>	
Material type	Reinforcement steel
Thermal expansion [m/mK]	0.00
Unit mass [kg/m <sup>3</sup> ]	7850.00
E modulus [MPa]	2.0000e+05
Poisson coeff.	0.2
Independent G modulus	□
G modulus [MPa]	8.3333e+04
Log. decrement	0.2
Colour	
Specific heat [J/gK]	6.0000e-01
Thermal conductivity [W/mK]	4.5000e+01
Bar surface	Ribbed
Order in code	2
<b>EN 1992-1-1</b>	
Characteristic yield strength $f_{yk}$ [MPa]	500.0
Calculated dependent values	□
Characteristic maximum tensile strength $f_{tk}$ [MPa]	525.0
Coefficient $k = f_{tk} / f_{yk}$ [-]	1.05
Design yield strength - persistent ( $f_{pd} = f_{yk} / \gamma_{s,p}$ ) [MPa]	434.8
Design yield strength - accidental ( $f_{pd} = f_{yk} / \gamma_{s,a}$ ) [MPa]	500.0
Maximum elongation $\epsilon_{ps,uk}$ [‰]	250.0
Class	A
Reinforcement type	Bars
Fabrication	Hot rolled
<b>Stress-strain diagram</b>	
Type of diagram	Bi-linear with an inclined top branch
Picture of Stress-strain diagram	

- Specified yield strength range:  $f_{yk} = 400$  to 600 MPa
- Specific properties for classes A – B – C: see Annex C

Product form		Bars and de-coiled rods			Wire Fabrics			Requirement or quantile value (%)
Class		A	B	C	A	B	C	-
Characteristic yield strength $f_{yk}$ or $f_{0,2k}$ (MPa)		400 to 600						5,0
Minimum value of $k = (f_t/f_y)_k$		$\geq 1,05$	$\geq 1,08$	$\geq 1,15$ $< 1,35$	$\geq 1,05$	$\geq 1,08$	$\geq 1,15$ $< 1,35$	10,0
Characteristic strain at maximum force, $\epsilon_{uk}$ (%)		$\geq 2,5$	$\geq 5,0$	$\geq 7,5$	$\geq 2,5$	$\geq 5,0$	$\geq 7,5$	10,0
Bendability		Bend/Rebend test			-			
Shear strength		-			0,3 A $f_{yk}$ (A is area of wire)			Minimum
Maximum deviation from nominal mass (individual bar or wire) (%)	Nominal bar size (mm)	$\pm 6,0$ $\pm 4,5$						5,0
	$\leq 8$							
	$> 8$							

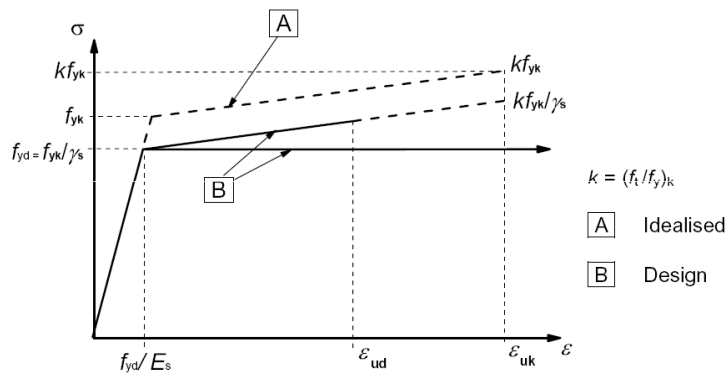
Class A is used by default; B and C have more rotation capacity.

### Stress-strain diagrams of typical reinforcing steel



- yield strength  $f_{yk}$  (or the 0,2% proof stress,  $f_{0,2k}$ )
- tensile strength  $f_{tk}$
- adequate ductility is necessary, defined by the  $(f_t/f_y)_k$  and  $\epsilon_{uk}$

### Stress-strain diagrams for design



Recommended value:

$$\epsilon_{ud} = 0,9 \epsilon_{uk}$$

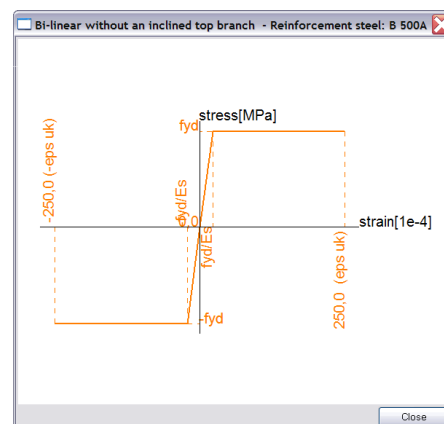
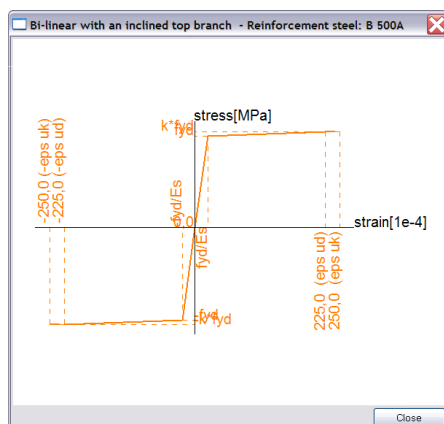
Assumptions for design:

- (a) inclined top branch with a strain limit of  $\epsilon_{ud}$  and a maximum stress of  $kf_{yk}/\gamma_s$  at  $\epsilon_{uk}$
- (b) horizontal top branch without the check of strain limit

## Section 3 Materials

### Stress-strain diagrams for design in Scia Engineer

<b>Stress-strain diagram</b>	
Type of diagram	Bi-linear with an inclined top branch
Picture of Stress-strain diagram	Bi-linear with an inclined top branch Bi-linear without an inclined top branch



## Section 4 Durability and cover to reinforcement

26

## Section 4 Durability and cover to reinforcement

Class designation	Description of the environment	Informative examples where exposure classes may occur
<b>1 No risk of corrosion or attack</b>		
X0	For concrete without reinforcement or embedded metal: all exposures except where there is freeze/thaw, abrasion or chemical attack For concrete with reinforcement or embedded metal: very dry	Concrete inside buildings with very low air humidity
<b>2 Corrosion induced by carbonation</b>		
XC1	Dry or permanently wet	Concrete inside buildings with low air humidity Concrete permanently submerged in water
XC2	Wet, rarely dry	Concrete surfaces subject to long-term water contact Many foundations
XC3	Moderate humidity	Concrete inside buildings with moderate or high air humidity External concrete sheltered from rain
XC4	Cyclic wet and dry	Concrete surfaces subject to water contact, not within exposure class XC2
<b>3 Corrosion induced by chlorides</b>		
XD1	Moderate humidity	Concrete surfaces exposed to airborne chlorides
XD2	Wet, rarely dry	Swimming pools Concrete components exposed to industrial waters containing chlorides
XD3	Cyclic wet and dry	Parts of bridges exposed to chlorides Pavements Car park slabs
<b>4 Corrosion induced by chlorides from sea water</b>		
XS1	Exposed to airborne salt but not in direct contact with sea water	Structures near to or on the sea
XS2	Permanently submerged	Parts of marine structures
XS3	Tidal, splash and spray zones	Parts of marine structures

**EN Table 4.1 Exposure classes related to environmental conditions**

<b>5. Freeze/Thaw Attack</b>		
XF1	Moderate water saturation, without de-icing agent	Vertical concrete surfaces exposed to rain and freezing
XF2	Moderate water saturation, with de-icing agent	Vertical concrete surfaces of road structures exposed to freezing and airborne de-icing agents
XF3	High water saturation, without de-icing agents	Horizontal concrete surfaces exposed to rain and freezing
XF4	High water saturation with de-icing agents or sea water	Road and bridge decks exposed to de-icing agents Concrete surfaces exposed to direct spray containing de-icing agents and freezing Splash zone of marine structures exposed to freezing
<b>6. Chemical attack</b>		
XA1	Slightly aggressive chemical environment according to EN 206-1, Table 2	Natural soils and ground water
XA2	Moderately aggressive chemical environment according to EN 206-1, Table 2	Natural soils and ground water
XA3	Highly aggressive chemical environment according to EN 206-1, Table 2	Natural soils and ground water

27



### Concrete cover

#### Nominal cover

$$c_{\text{nom}} = c_{\text{min}} + \Delta c_{\text{dev}} \quad (4.1)$$

$c_{\text{min}}$ , minimum cover, in order to ensure:

- the safe transmission of bond forces
- the protection of the steel against corrosion (durability)
- an adequate fire resistance (see EN 1992-1-2)

$\Delta c_{\text{dev}}$ , allowance in design for deviation

### Concrete cover

#### Minimum cover, $c_{\text{min}}$

$$c_{\text{min}} = \max \{ c_{\text{min,b}} ; c_{\text{min,dur}} + \Delta c_{\text{dur,g}} - \Delta c_{\text{dur,st}} - \Delta c_{\text{dur,add}} ; 10 \text{ mm} \} \quad (4.2)$$

where:

- $c_{\text{min,b}}$  minimum cover due to bond requirement
- $c_{\text{min,dur}}$  minimum cover due to environmental conditions
- $\Delta c_{\text{dur,g}}$  additive safety element
- $\Delta c_{\text{dur,st}}$  reduction of minimum cover for use of stainless steel
- $\Delta c_{\text{dur,add}}$  reduction of minimum cover for use of additional protection

#### Allowance in design for deviation, $\Delta c_{\text{dev}}$

$$\Delta c_{\text{dev}} = 10 \text{ mm} \quad (\text{recommended value})$$

Concrete cover in Scia Engineer

EC-EN

- Concrete
  - Design defaults
    - Concrete cover**
    - Columns
    - Beams
    - 2D structures and beam slabs
    - Punching
    - Default sway type (for columns and beams)
  - General
    - Concrete
    - Non-reinforced reinforcement

Name	EC-EN
Concrete	
Design defaults	
Concrete cover	
Use min concrete cover	<input checked="" type="checkbox"/>
Design working life (years)	50
Exposure class	XC3
Abrasion class	None
Type of concrete	In-situ concrete
Special quality control	<input type="checkbox"/> no

Exposure class	XC3
Abrasion class	X0
Type of concrete	XC1
Special quality control	XC2
Columns	XC3
Beams	XC4
2D structures and beam slabs	XD1
Punching	XD2
Default sway type (for columns and beams)	XD3
General	XS1
	XS2
	XS3

Abrasion class	None
Type of concrete	None
Special quality control	XM1
Columns	XM2
Beams	XM3

Section 5 Structural Analysis

### Structural models for overall analysis

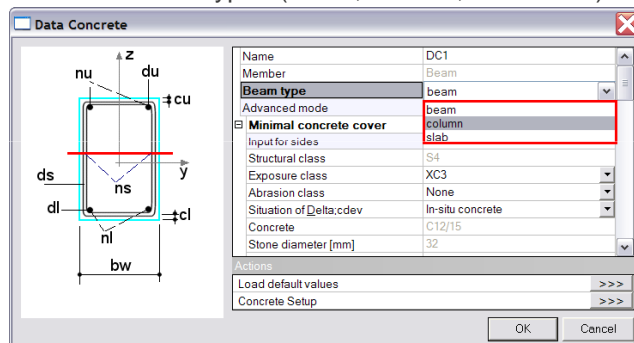
The elements of a structure are classified, by consideration of their nature and function, as **beams, columns, slabs, walls, plates, arches, shells** etc. Rules are provided for the analysis of the commoner of these elements and of structures consisting of combinations of these elements.

See EN § 5.3.1 for the descriptions

## Section 5 Structural Analysis

### Assignment of structural models in Scia Engineer

- For **1D members**: 3 types (beam, column, beam slab) – choice made by user

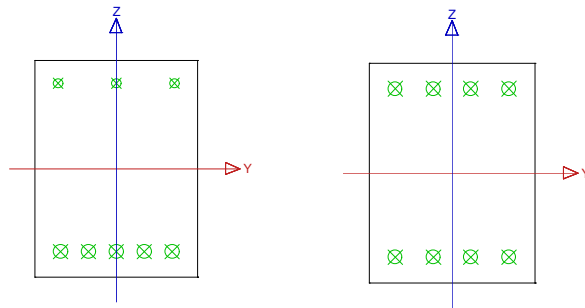


! Different calculation methods !

- For **2D members**: 3 types (plate, wall, shell) – detected by NEDIM solver, based on present internal forces

### Assignment of structural models in Scia Engineer

Beam calculation ↔ Column calculation



Difference in reinforcement area per direction

Internal forces taken into account:

- Beam calculation:  $N$ ,  $M_y$ ,  $V_z$
- Column calculation:  $N$ ,  $M_y$ ,  $M_z$ ,  $V_z$

## Section 5 Structural Analysis

### Geometric data

Continuous slabs and beams may generally be analysed on the assumption that the supports provide no rotational restraint.

- **Monolithic connection (Fixed support)**

Critical design moment at the support = Moment at the face of the support

- **Hinged support**

Design support moment may be reduced by an amount  $\Delta M_{Ed}$ :

$$\Delta M_{Ed} = F_{Ed,sup} t / 8 \quad (5.9)$$

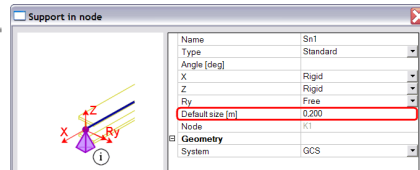
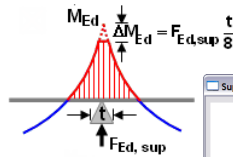
where:  $F_{Ed,sup}$  is the design support reaction  
 $t$  is the breadth of the support

### Moment reduction above support in Scia Engineer

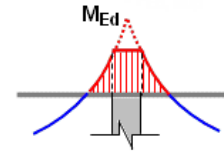
- Durability and concrete cover
- Calculation
  - General
  - Columns
  - Beams
- ULS
  - General
  - Interaction diagram

Beams	
Calculate compression reinforcement	<input checked="" type="checkbox"/> yes
Include normal force to calculation	<input checked="" type="checkbox"/> yes
Check compression of member	<input type="checkbox"/> no
$NEd < x'Ac'f_{cd}$ $x = [-]$	0.00
Moment reduction at supports	<input type="checkbox"/> no
Shear force reduction at supports	<input type="checkbox"/> no

Nodal support



Column support



## Section 5 Structural Analysis

Common **idealisations of the behaviour** used for analysis are:

- (a) linear elastic behaviour
- (b) linear elastic behaviour with limited redistribution
- (c) plastic behaviour, including strut and tie models
- (d) non-linear behaviour

- Based on the theory of elasticity
- Suitable for both ULS and SLS
- Assumptions:
  - uncracked cross-sections
  - linear stress-strain relationships
  - mean value of  $E$
- For thermal deformation, settlement and shrinkage effects:
  - at ULS: reduced stiffness (cracking - tension stiffening + creep)
  - at SLS: gradual evolution of cracking

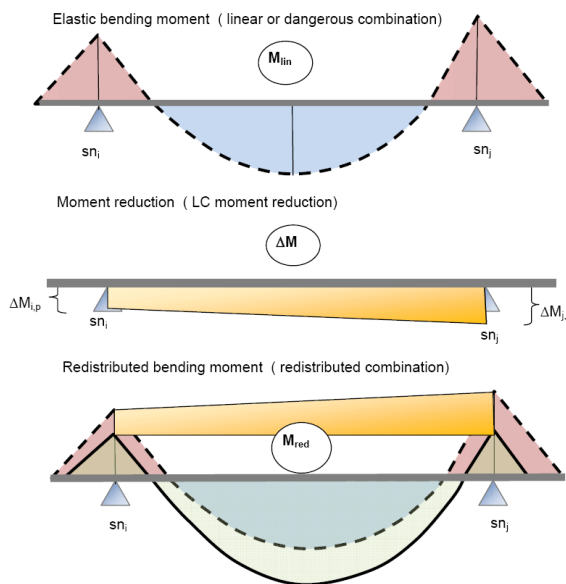
### Linear elastic analysis in Scia Engineer

Linear calculation

38

## Section 5 Structural Analysis

### Principle of redistribution of bending moment



### Application

For analysis of structural members for the verification of ULS.

39

- The moments at ULS calculated using a linear elastic analysis may be redistributed, provided that the resulting distribution of moments remains in **equilibrium** with the applied loads.
- Redistribution of bending moments, without explicit check on the rotation capacity, is allowed in continuous beams or slabs provided that:
  - they are predominantly subject to flexure
  - the ratio of the lengths of adjacent spans is in the range of 0,5 to 2
  - $\delta \geq k_1 + k_2 x_u/d$  for  $f_{ck} \leq 50$  MPa (5.10a)
  - $\delta \geq k_3 + k_4 x_u/d$  for  $f_{ck} > 50$  MPa (5.10b)
    - $\geq k_5$  where Class B and Class C reinforcement is used (see EN Annex C)
    - $\geq k_6$  where Class A reinforcement is used (see EN Annex C)

where:

$\delta$  is the ratio of the redistributed moment to the elastic bending moment

$x_u$  is the depth of the neutral axis at the ultimate limit state after redistribution

$d$  is the effective depth of the section

$k_1, k_2, k_3, k_4, k_5$  and  $k_6$ : see recommended values in National Annex

- Redistribution should not be carried out in circumstances where the rotation capacity cannot be defined with confidence.
- For the design of columns the elastic moments from frame action should be used without any redistribution.

### Plastic analysis for beams, frames and slabs

- Only suitable for ULS
- Plastic analysis without any direct check of rotation capacity may be used, if the ductility of the critical sections is sufficient for the envisaged mechanism to be formed.
- The required ductility may be deemed to be satisfied without explicit verification if all the following are fulfilled:
  - the area of tensile reinforcement is limited such that, at any section
$$x_u/d \leq 0,25 \text{ for concrete strength classes } \leq C50/60$$
$$x_u/d \leq 0,15 \text{ for concrete strength classes } \geq C55/67$$
  - reinforcing steel is either Class B or C (see EN Annex C)
  - the ratio of the moments at intermediate supports to the moments in the span is between 0,5 and 2

### Options for redistribution of bending moments & plastic analysis in Scia Engineer

Available since Scia Engineer 2010.0

Choice for desired method in the Concrete Setup:

<input checked="" type="checkbox"/> <b>Check redistributed moments</b>	
Check acc to 5.5(4)	<input type="checkbox"/> no
Check acc to 5.6.2(2)	<input type="checkbox"/> no
Check rotation capacity 5.6.3	<input type="checkbox"/> no

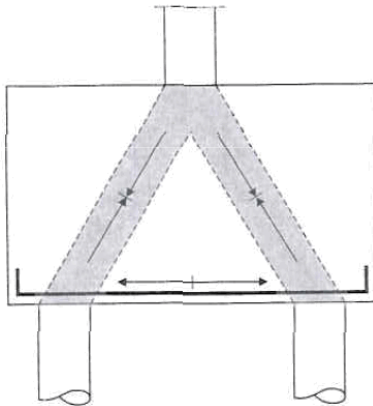


### Analysis with strut-and-tie models

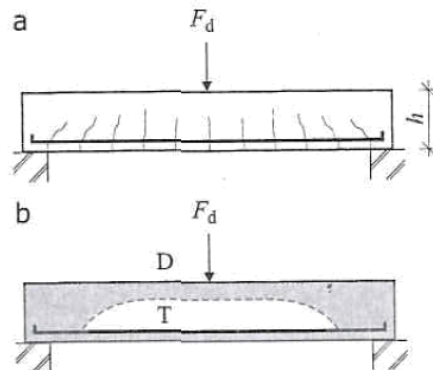
- Strut-and-tie models consist of
  - struts = compressive stress fields
  - ties = reinforcement
  - connecting nodes
- Forces: determined by maintaining the equilibrium with the applied loads in the ULS

## Section 5 Structural Analysis

### Analysis with strut and tie models: Comparison deep beam ↔ slender beam



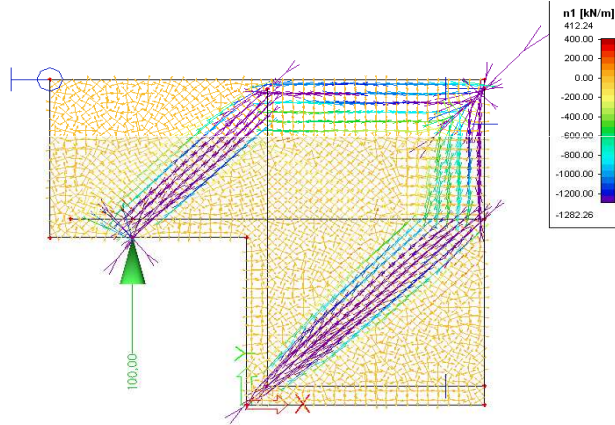
Deep beam  
with pressure diagonals & tension tie



Slender beam  
with pressure arch (D) & tension tie (T)

### Analysis with strut and tie models in Scia Engineer

Pressure only 2D members – Trajectories result



46

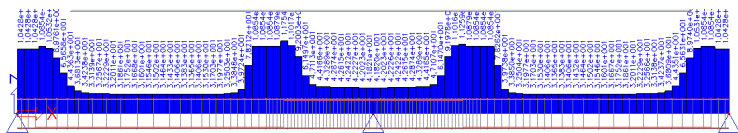
## Section 5 Structural Analysis

- Suitable for both ULS and SLS, provided that equilibrium and compatibility are satisfied
- Non-linear behaviour for materials taken into account  
See EN Section 3 for the non-linear  $\sigma$ - $\epsilon$  diagrams
- The analysis may be first or second order

### Non-linear analysis in Scia Engineer

1<sup>st</sup> order: PNL calculation

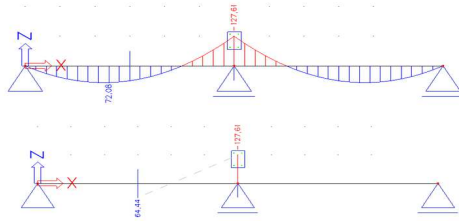
2<sup>nd</sup> order: PGNL calculation



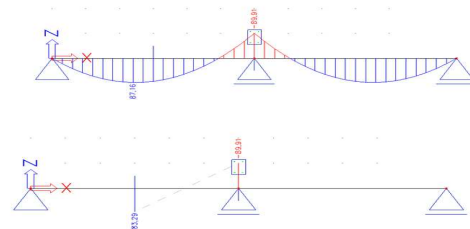
47

### Non-linear analysis in Scia Engineer: Plastic hinges

Linear calculation



Non-linear calculation



Change of the stiffness in the concrete cross-section above the middle support due to cracks. The **plastic resistance moment** is reached.

→ Redistribution of the moment line to satisfy the equilibrium of the structure

Principle: Non-linear moment above the support + ½ Non-linear moment at the half of the span = Linear moment above the support

## Section 5 Structural Analysis

### Definition of Geometric imperfections

- As consideration of the unfavourable effects of possible deviations in the geometry of the structure and the position of loads.

(Deviations in cross-section dimensions → material safety factors)

- To be taken into account both in 1<sup>st</sup> and 2<sup>nd</sup> order calculation.
- To be taken into account only in ULS (persistent and accidental design situations).
- To be taken into account only in the direction where they will have the most unfavourable effect.

### General

Imperfections may be represented by an **inclination  $\theta_i$** :

$$\theta_i = \theta_0 \alpha_h \alpha_m \quad (5.1)$$

where:

$\theta_0$  is the basic value = 1/200 (recommended value)

$\alpha_h$  is the reduction factor for length or height

$\alpha_m$  is the reduction factor for number of vertical members contributing to the total effect

## Section 5 Structural Analysis

### Isolated members

2 alternative ways to take geometric imperfections into account

(1) As an **eccentricity,  $e_i$** , given by

$$e_i = \theta_i l_0 / 2 \quad (5.2)$$

where  $l_0$  is the effective length

For walls and isolated columns in braced systems,  $e_i = l_0 / 400$  may always be used as a simplification, corresponding to  $\alpha_h = 1$ .

(2) As a **transverse force,  $H_i$** , in the position that gives the maximum moment:

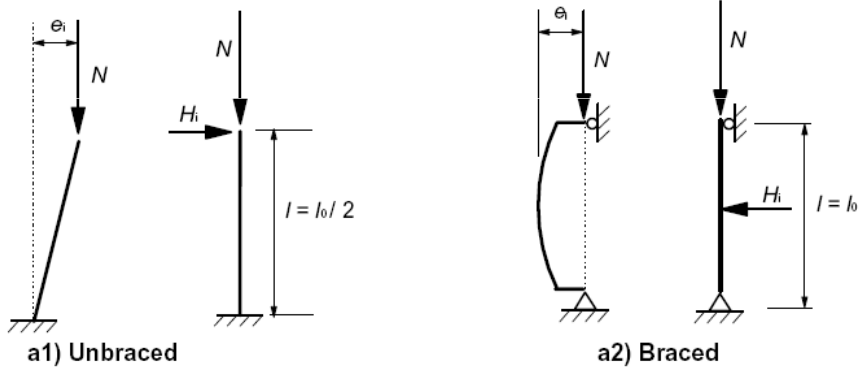
$$\text{- for unbraced members: } H_i = \theta_i N \quad (5.3a)$$

$$\text{- for braced members: } H_i = 2 \theta_i N \quad (5.3b)$$

where  $N$  is the axial load

### Isolated members

2 alternative ways to take geometric imperfections into account



## Section 5 Structural Analysis

### Minimum eccentricity for cross-section design

$$e_{0,min} = \max \{ h/30 ; 20 \text{ mm} \}$$

see EN § 6.1(4)

where  $h$  is the depth of the section

This means 
$$e_0 = \max \left[ (e_1 + e_i); \frac{h}{30}; 20 \text{ mm} \right]$$

where:

$e_1$  = 1<sup>st</sup> order eccentricity

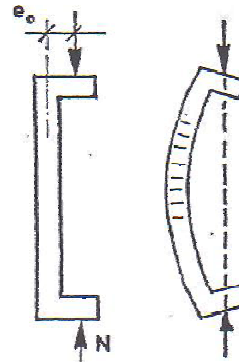
$e_i$  = eccentricity due to geometric imperfections

$e_0 = e_1 + e_i$ , design eccentricity in a 1<sup>st</sup> order calculation

### Definitions

- *First order effects*: action effects calculated without consideration of the effect of structural deformations, but including geometric imperfections.
- *Second order effects*: additional action effects caused by structural deformations.

They shall be taken into account where they are likely to affect the overall stability of a structure significantly; e.g. in case of columns, walls, piles, arches and shells.



### Criteria for 1<sup>st</sup> or 2<sup>nd</sup> order calculation

2<sup>nd</sup> order effects have to be taken into account in each direction, unless they may be ignored acc. to *one* of the following articles:

(a) 2<sup>nd</sup> order effects may be ignored if they are less than 10 % of the corresponding 1<sup>st</sup> order effects.

(b) Simplified criterion = **Slenderness criterion** for isolated members

2<sup>nd</sup> order effects may be ignored if the slenderness  $\lambda < \lambda_{lim}$

In case of biaxial bending:

the slenderness criterion may be checked separately for each direction

### Criteria for 1<sup>st</sup> or 2<sup>nd</sup> order calculation

#### Slenderness ratio $\lambda$

$$\lambda = l_0 / i \quad (5.14)$$

where:

$l_0$  is the effective length

$i$  is the radius of gyration of the uncracked concrete section

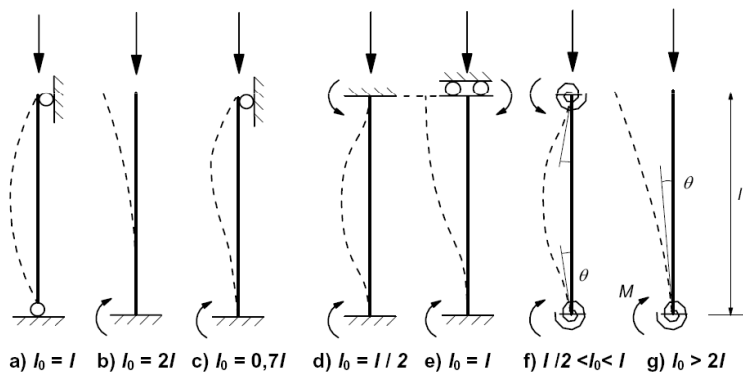
#### Limit slenderness $\lambda_{lim}$

$$\lambda_{lim} = 20 A B C / \sqrt{n} \quad (\text{recommended value, 5.13N})$$

## Section 5 Structural Analysis

### Criteria for 1<sup>st</sup> or 2<sup>nd</sup> order calculation

#### Effective length - for isolated members



### Methods of analysis - Taking 2<sup>nd</sup> order effects into account

- General method: based on non-linear 2<sup>nd</sup> order analysis
- Two simplified methods: based on linear (nominal 2<sup>nd</sup> order) analysis
  - (a) Method based on nominal stiffness & moment magnification factor
    - Use: both isolated members and whole structures
  - (b) Method based on nominal curvature
    - Use: mainly suitable for isolated members, but with realistic assumptions concerning the distribution of curvature, also for structures
- The selection of simplified method (a) and (b) to be used in a Country may be found in its National Annex.

### General method

#### General method in Scia Engineer

Real (physical and) geometrical non-linear calculation

- PGNL analysis for 1D members:
  - non-linear  $\sigma$ - $\epsilon$  diagram, new stiffness EI is calculated iteratively
- GNL analysis for 2D members:
  - no non-linear  $\sigma$ - $\epsilon$  diagram,
  - but approximation of new stiffness EI by adapting value of E in the material library:

$$EI = K_c E_{cd} I_c + K_s E_s I_s \quad \text{OR} \quad E_{cd,eff} = \frac{E_{cd}}{1 + \varphi_{eff}}$$



### Simplified methods

#### (a) Method based on nominal stiffness

##### Nominal stiffness

$$EI = K_c E_{cd} I_c + K_s E_s I_s \quad E_{cd,eff} = \frac{E_{cd}}{1 + \varphi_{ef}}$$

##### Moment magnification factor

Total moment = 1<sup>st</sup> and 2<sup>nd</sup> order moment

$$M_{ed} = M_{0Ed} \left[ 1 + \frac{\beta}{(N_B/N_{Ed}) - 1} \right]$$

#### (b) Method based on nominal curvature

##### Application

For isolated members with constant normal force  $N$  and a defined effective length  $l_0$ .

The method gives a nominal 2<sup>nd</sup> order moment based on a deflection, which in turn is based on the effective length and an estimated maximum curvature.

##### Design moment $M_{Ed}$

$$M_{Ed} = M_{0Ed} + M_2 \quad (5.31)$$

where:

$M_{0Ed}$  is the 1<sup>st</sup> order moment, including the effect of imperfections

$M_2$  is the nominal 2<sup>nd</sup> order moment, based on 1<sup>st</sup> order internal forces

### (b) Method based on nominal curvature

Nominal 2<sup>nd</sup> order moment  $M_2$

$$M_2 = N_{Ed} e_2 \quad (5.33)$$

where:

$N_{Ed}$  is the design value of axial force

$e_2$  is the deflection =  $(1/r) * (l_o^2/c)$

$1/r$  is the curvature

$l_o$  is the effective length

$c$  is a factor depending on the curvature distribution

For constant cross-section,  $c = 10 (\approx \pi^2)$  is normally used. If the first order moment is constant, a lower value should be considered (8 is a lower limit, corresponding to constant total moment).

### (b) Method based on nominal curvature

Curvature  $1/r$

$$1/r = K_r K_\varphi 1/r_0 \quad (5.34)$$

where:

$K_r$  is a correction factor depending on axial load

$K_\varphi$  is a factor for taking account of creep

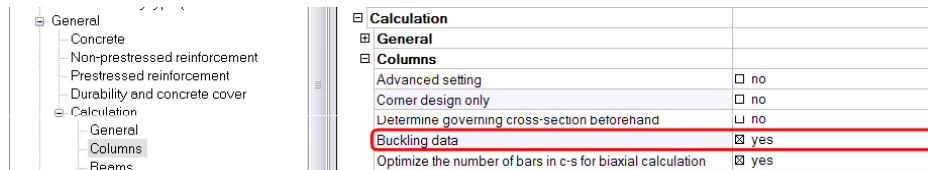
$1/r_0 = \varepsilon_{yd} / (0,45 d)$

$\varepsilon_{yd} = f_{yd} / E_s$

$d$  is the effective depth

### (b) Method based on nominal curvature in Scia Engineer

Linear calculation



Taken into account for design:

"Buckling data" OFF: - 1<sup>st</sup> order moment

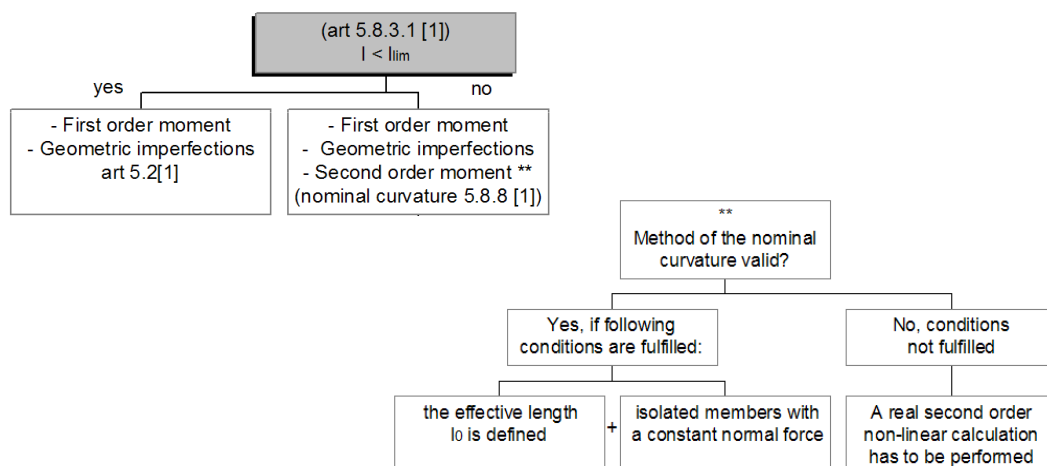
"Buckling data" ON:

- 1<sup>st</sup> order moment
- moment caused by geometrical imperfections
- nominal 2<sup>nd</sup> order moment, only if  $\lambda > \lambda_{lim}$

## Section 5 Structural Analysis

### (b) Method based on nominal curvature in Scia Engineer

Overview

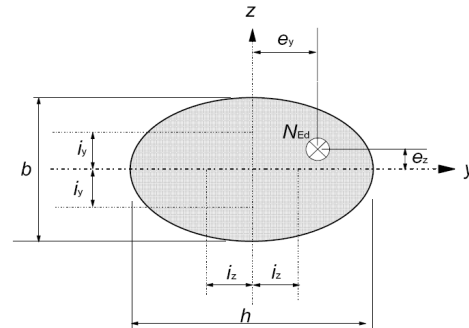


### Bi-axial bending

To decide if a bi-axial bending calculation is required or not, the following conditions should be checked: (slenderness ratios & relative eccentricities)

$$\frac{\lambda_y}{\lambda_z} \leq 2 \quad \text{and} \quad \frac{\lambda_z}{\lambda_y} \leq 2$$

$$\frac{e_{Ed,y} / b_{eq}}{e_{Ed,z} / h_{eq}} \leq 0,2 \quad \text{or} \quad \frac{e_{Ed,z} / h_{eq}}{e_{Ed,y} / b_{eq}} \leq 0,2$$



If these conditions are NOT fulfilled → bi-axial bending calculation is required

## Section 5 Structural Analysis

### Biaxial bending

Simplified criterion = Interaction formula

$$\left( \frac{M_{Edz}}{M_{Rdz}} \right)^a + \left( \frac{M_{Edy}}{M_{Rdy}} \right)^a \leq 1 \quad (5.39)$$

where:

$M_{Edz/y}$  is the design moment around the respective axis, including a 2<sup>nd</sup> order moment (if required)

$M_{Rdz/y}$  is the moment resistance in the respective direction

a is the exponent; for circular and elliptical cross sections: a = 2;

for rectangular cross sections:

$N_{Ed}/N_{Rd}$	0,1	0,7	1,0
a =	1,0	1,5	2,0

with linear interpolation for intermediate values

### Column calculation in Scia Engineer

<b>Calculation Method</b>	
Type of calculation method	Automatic determination
Automatic determination - Uni-axial b...	Uni-axial bending calculation (sum)
<b>Design reinforcement by using (...)</b>	Uni-axial bending calculation (max)
Area of reinforcement type	Bi-axial bending calculation (interaction formula)
Delta area of reinforcement [mm^2]	Automatic determination

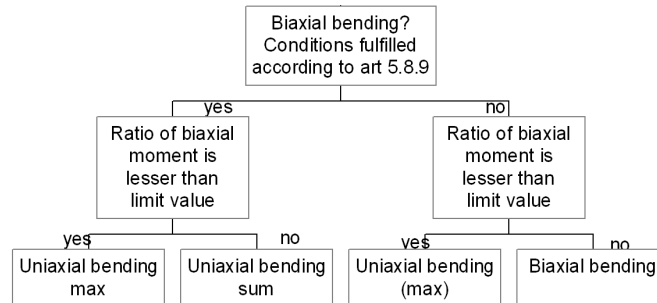
**Conditions 5.8.9(3)**

$$\frac{\lambda_y}{\lambda_z} \leq 2 \quad \text{and} \quad \frac{\lambda_z}{\lambda_y} \leq 2$$

$$\frac{e_{Ed,y} / b_{eq}}{e_{Ed,z} / h_{eq}} \leq 0,2 \quad \text{or} \quad \frac{e_{Ed,z} / h_{eq}}{e_{Ed,y} / b_{eq}} \leq 0,2$$

**Ratio of biaxial moments**

$$\frac{\min(|M_{Ed,y}|, |M_{Ed,z}|)}{\max(|M_{Ed,y}|, |M_{Ed,z}|)} \cdot 100 \leq 10\%$$



## Section 6 Ultimate limit states (ULS)

### Application

Section 6 applies to undisturbed regions of beams, slabs and similar types of members for which sections remain approximately plane before and after loading.

If plane sections do not remain plane → see EN § 6.5 (Design with strut and tie models)

### Ultimate moment resistance $M_{Rd}$ (or $M_u$ ) of reinforced concrete cross-sections

Assumptions when determining  $M_{Rd}$ :

- plane sections remain plane
- strain in bonded reinforcement = strain in the surrounding concrete
- tensile strength of concrete is ignored
- stresses in concrete in compression → see design  $\sigma$ - $\epsilon$  diagrams (EN § 3.1.7)
- stresses in reinforcing steel → see design  $\sigma$ - $\epsilon$  diagrams (EN § 3.2.7)

## Section 6 Ultimate limit states (ULS)

### Strain limits

- Reinforcing steel                      Tensile strain limit =  $\epsilon_{ud}$  (where applicable)

- Concrete

- In sections mainly subjected to bending

Compressive strain limit =  $\epsilon_{cu2}$  (or  $\epsilon_{cu3}$ )

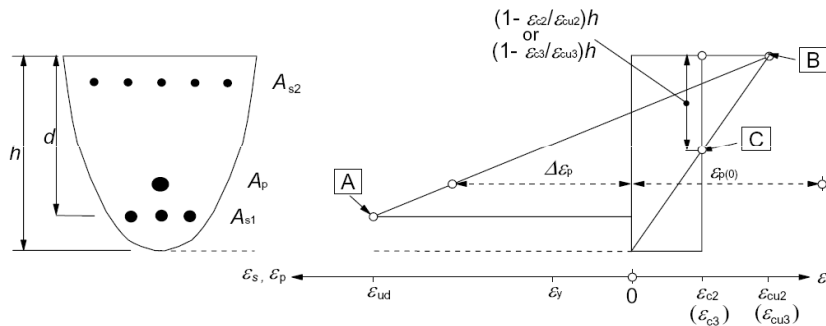
- In sections subjected to  $\pm$  pure compression  
( $\pm$  concentric loading ( $e_d/h < 0,1$ ), e.g. compression flanges of box girders, columns, ...)

Pure compressive strain limit =  $\epsilon_{c2}$  (or  $\epsilon_{c3}$ )

For concentrically loaded cross-sections with symmetrical reinforcement, assume as eccentricity  $e_0 = \max \left[ (e_1 + e_i); \frac{h}{30}; 20\text{mm} \right]$  ( $M_{Ed}$  is at least =  $e_0 N_{Ed}$ )

where  $h$  is the depth of the section

### Possible range of strain distributions (ULS)



- A** - reinforcing steel tension strain limit
- B** - concrete compression strain limit
- C** - concrete pure compression strain limit

### General verification procedure

#### Definitions

- $V_{Ed}$  = design shear force resulting from external loading
- $V_{Rd,c}$  = design shear resistance of the member *without* shear reinforcement
- $V_{Rd,s}$  = design value of the shear force which can be sustained by the yielding shear reinforcement
- $V_{Rd,max}$  = design value of the maximum shear force which can be sustained by the member, limited by crushing of the concrete compression struts

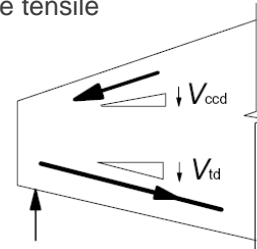
### General verification procedure

#### Definitions

In members with inclined chords the following additional values are defined:

$V_{ccd}$  = design value of the shear component of the force in the compression area, in the case of an inclined compression chord

$V_{td}$  = design value of the shear component of the force in the tensile reinforcement, in the case of an inclined tensile chord



Shear resistance of a member with shear reinforcement:

$$V_{Rd} = V_{Rd,s} + V_{ccd} + V_{td} \quad (6.1)$$

### General verification procedure

#### Overview

If  $V_{Ed} \leq V_{Rd,c}$  No shear reinforcement required (theoretically), but minimum shear reinforcement should be provided for beams:

$$\rho_{w,min} = (0,08 \sqrt{f_{ck}}) / f_{yk} \quad (\text{recommended value, 9.5N})$$

If  $V_{Ed} > V_{Rd,c}$  Shear reinforcement should be provided in order that  $V_{Ed} \leq V_{Rd}$

$$\text{In practice: } V_{Rd,s} = V_{Ed} - V_{ccd} - V_{td}$$

$$\text{Check if } V_{Rd,s} \text{ (or } V_{Ed}) \leq V_{Rd,max}$$

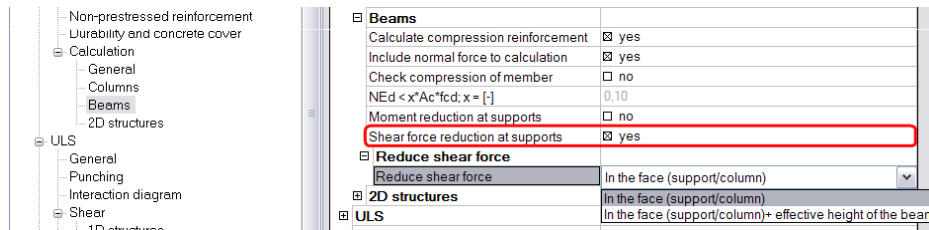
(If  $V_{Ed} > V_{Rd,max}$ , failure by crushing of concrete compression struts!)



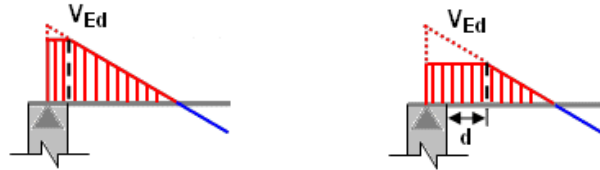
### Shear force reduction at supports

For members subject to predominantly uniformly distributed loading, the design shear force need not to be checked at a distance less than  $d$  from the face of the support. Any shear reinforcement required should continue to the support.

#### ... in Scia Engineer



2 options:



76

## Section 6 Ultimate limit states (ULS)

### $V_{Ed} \leq V_{Rd,c}$ : Members not requiring design shear reinforcement

Design value for the shear resistance  $V_{Rd,c}$

$$V_{Rd,c} = [C_{Rd,c} k (100 \rho_l f_{ck})^{1/3} + k_1 \sigma_{cp}] b_w d \quad (6.2a)$$

$$\text{with a minimum of } V_{Rd,c} = (v_{min} + k_1 \sigma_{cp}) b_w d \quad (6.2b)$$

where:

$$k = 1 + \sqrt{(200/d)} \leq 2,0 \text{ with } d \text{ in [mm]}$$

$$\rho_l = A_{sl} / b_w d \leq 0,02 \text{ is the longitudinal reinforcement ratio}$$

$A_{sl}$  is the area of the tensile reinforcement, which extends  $\geq (l_{bd} + d)$  beyond the section considered

$b_w$  is the smallest width of the cross-section in the tensile area [mm]

$$\sigma_{cp} = N_{Ed} / A_c < 0,2 f_{cd} \text{ [MPa] with } N_{Ed} > 0 \text{ for compression}$$

$$C_{Rd,c} = 0,18 / \gamma_c \quad (\text{recommended value})$$

$$v_{min} = 0,035 k^{3/2} \cdot f_{ck}^{1/2} \quad (\text{recommended value, 6.3N})$$

$$k_1 = 0,15 \quad (\text{recommended value})$$

77

### $V_{Ed} \leq V_{Rd,c}$ : Members not requiring design shear reinforcement

$V_{Ed}$  should always satisfy the condition

$$V_{Ed} \leq 0,5 b_w d \nu f_{cd} \quad (6.5)$$

where:

$\nu$  is a strength reduction factor for concrete cracked in shear

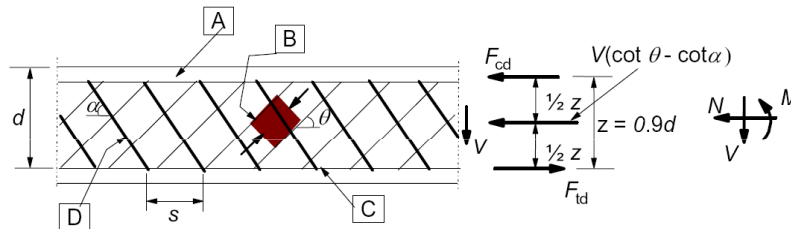
$$\nu = 0,6 [1 - (f_{ck}/250)] \quad (\text{recommended value, } 6.6N)$$

with  $f_{ck}$  in [MPa]

## Section 6 Ultimate limit states (ULS)

### $V_{Ed} > V_{Rd,c}$ : Members requiring design shear reinforcement

The design of members with shear reinforcement is based on a **truss model**



[A] - compression chord, [B] - struts, [C] - tensile chord, [D] - shear reinforcement

$\alpha$  = angle between the shear reinforcement and the beam axis

$\theta$  = angle between the concrete compression strut and the beam axis

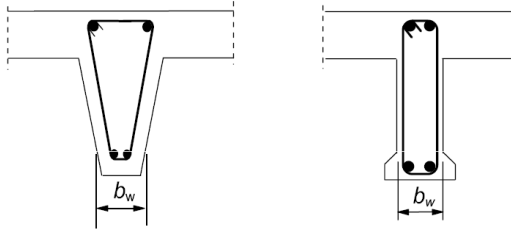
$$1 \leq \cot \theta \leq 2,5 \quad (\text{recommended limits, } 6.7N)$$

$F_{td}$  = design value of the tensile force in the longitudinal reinforcement

$F_{cd}$  = design value of the concrete compression force

$z$  = the inner lever arm; the approximate value  $z = 0,9 d$  may normally be used

### $V_{Ed} > V_{Rd,c}$ : Members requiring design shear reinforcement



$b_w$  = minimum width between tension and compression chords

### $V_{Ed} > V_{Rd,c}$ : Members requiring design shear reinforcement

$\alpha = 90^\circ$  (vertical shear reinforcement)

$V_{Rd}$  is the smaller value of:

$$V_{Rd,s} = (A_{sw}/s) z f_{ywd} \cot\theta \quad (6.8)$$

and  $V_{Rd,max} = \alpha_{cw} b_w z \nu_1 f_{cd} / (\cot\theta + \tan\theta) \quad (6.9)$

where:

$A_{sw}$  is the cross-sectional area of the shear reinforcement

$s$  is the spacing of the stirrups

$f_{ywd}$  is the design yield strength of the shear reinforcement

$\nu_1$  is a strength reduction factor for concrete cracked in shear

$\alpha_{cw}$  is a coefficient taking account of the state of the stress in the compression chord

### $V_{Ed} > V_{Rd,c}$ : Members requiring design shear reinforcement

#### $\alpha = 90^\circ$ (vertical shear reinforcement)

$$v_1 = v \quad (\text{recommended value, } 6.6N)$$

If  $\sigma_{wd} < 0,80 f_{ywk}$ ,  $v_1$  may be taken as:

$$v_1 = 0,6 \quad \text{for } f_{ck} \leq 60 \text{ MPa} \quad (6.10.aN)$$

$$v_1 = 0,9 - f_{ck}/200 > 0,5 \quad \text{for } f_{ck} \geq 60 \text{ MPa} \quad (6.10.bN)$$

If this Expression (6.10) is used, the value of  $f_{ywd}$  should be reduced to  $0,80 f_{ywk}$  in Expression (6.8).

$$\alpha_{cw} = 1 \quad \text{for non-prestressed structures} \quad (\text{recommended value})$$

$$A_{sw,max}, \text{ for } \theta = 45^\circ \quad (\cot\theta = 1 \text{ and } \tan\theta = 1), \text{ and } V_{Rd,s} = V_{Rd,max}$$

$$A_{sw,max}/s = 0,5 \alpha_{cw} b_w v_1 f_{cd} / f_{ywd} \quad (6.12)$$

### $V_{Ed} > V_{Rd,c}$ : Members requiring design shear reinforcement

#### $\alpha < 90^\circ$ (inclined shear reinforcement)

$V_{Rd}$  is the smaller value of:

$$V_{Rd,s} = (A_{sw}/s) z f_{ywd} (\cot\theta + \cot\alpha) \sin\alpha \quad (6.13)$$

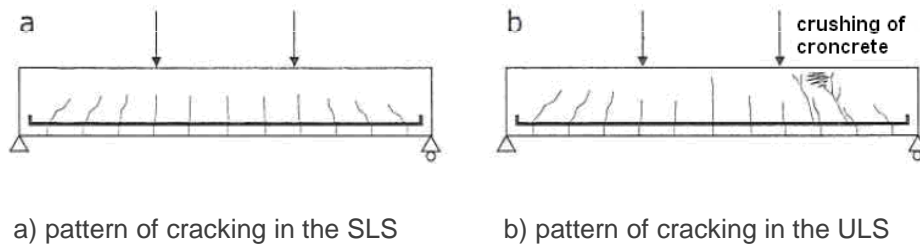
$$\text{and } V_{Rd,max} = \alpha_{cw} b_w v_1 f_{cd} (\cot\theta + \cot\alpha) / (1 + \cot^2\theta) \quad (6.14)$$

$$A_{sw,max}, \text{ for } \theta = 45^\circ \quad (\cot\theta = 1), \text{ and } V_{Rd,s} = V_{Rd,max}$$

$$A_{sw,max}/s = (0,5 \alpha_{cw} b_w v_1 f_{cd}) / (f_{ywd} \sin\alpha) \quad (6.15)$$

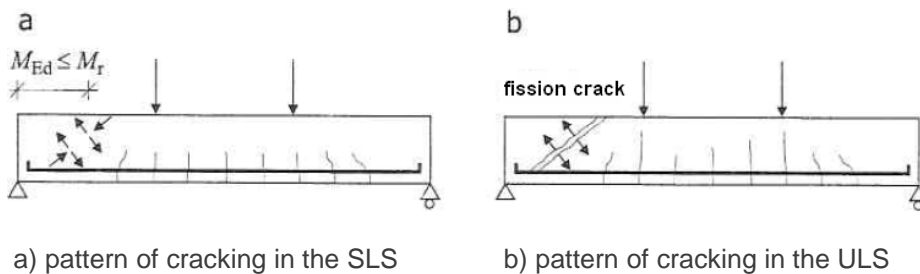
### Explanation for $V_{Rd,max}$ – Failure modes in case of shear

#### (1) Shear-bending failure



### Explanation for $V_{Rd,max}$ – Failure modes in case of shear

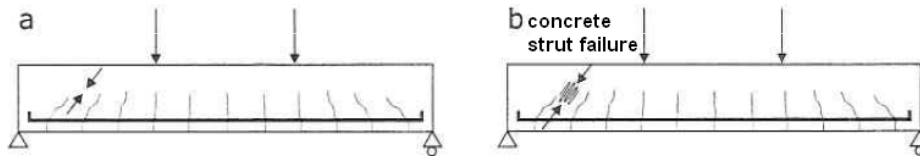
#### (2) Shear-tension failure



→ Adding enough shear reinforcement (in the form of vertical stirrups) prevents shear-bending and shear-tension failure.

### Explanation for $V_{Rd,max}$ – Failure modes in case of shear

#### (3) Shear-compression failure



a) pattern of cracking in the SLS

b) pattern of cracking in the ULS

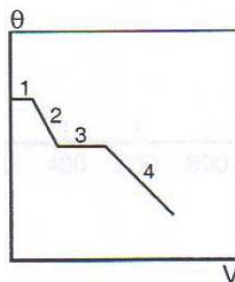
→ Imposing a maximum value  $V_{Rd,max}$  to the shear force  $V_{Ed}$  prevents sudden failure of the concrete compression strut before yielding of the shear reinforcement.

## Section 6 Ultimate limit states (ULS)

### Variable strut inclination method

The strut inclination  $\theta$  may be chosen between two limit values

$$1 \leq \cot \theta \leq 2,5 \quad \text{or} \quad 21,8^\circ < \theta < 45^\circ \quad (\text{recommended limits, 6.7N})$$



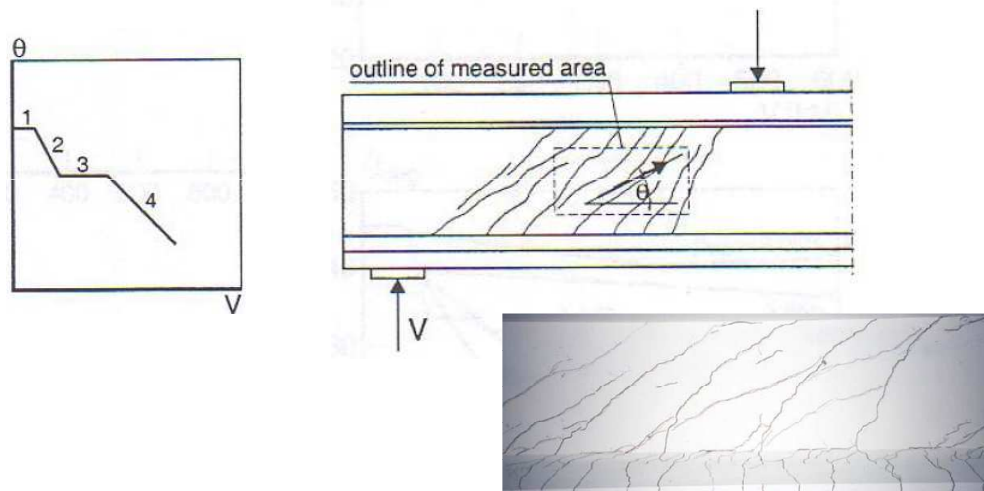
1) Web uncracked in shear

2) Inclined cracks appear

3) Stabilization of inclined cracks

4) Yielding of stirrups → further rotations and new cracks under lower angle → finally failure by web crushing

### Variable strut inclination method



88

### Variable strut inclination method

Advantages of this method:

Large freedom of design because of the large interval for  $\theta$

By making a good choice for the inclination of the struts, optimal design can be achieved:

- Larger angle  $\theta$  → higher value of  $V_{Rd,max}$  (saving on concrete)
- Smaller angle  $\theta$  → larger stirrup spacing is sufficient = smaller value of  $A_{sw}$  (saving on steel)

89

### Variable strut inclination method

General procedure – which can be used in Scia Engineer:

- Assume  $\theta = 21,8^\circ$  ( $\cot \theta = 2,5$ ) and calculate  $A_{sw}$
- Check if  $V_{Ed} > V_{Rd,max}$ : If NO → OK, end of design  
If YES → crushing of the concrete strut
- 3 options if  $V_{Rd,max}$  is exceeded:
  - increase height of beam
  - choose higher concrete class
  - increase  $\theta$ , or calculate  $\theta$  for which  $V_{Ed} = V_{Rd,s}$

and repeat the procedure

### Variable strut inclination method

User input of angle  $\theta$  (or cotangent  $\theta$ ) in Scia Engineer

Shear	
1D structures	
Shear coefficients	
Distance with full resistance from outside stirrup (multiple ...)	0,50
Angle between the concrete compression strut a...	
Type of input theta	Angle
Web	
theta [deg]	Angle
cot(theta)	Cotangent
	1,192
Compression flange	
theta [deg]	40,00
cot(theta)	1,192
Tension flange	
theta [deg]	40,00
cot(theta)	1,192



### Additional tensile force in the longitudinal reinforcement ... caused by shear

#### 2 approaches

(1) EN Section 6:

→ For members with shear reinforcement

Calculation of the **additional tensile force**,  $\Delta F_{td}$ , in the longitudinal reinforcement due to shear  $V_{Ed}$ :

$$\Delta F_{td} = 0,5 V_{Ed} (\cot\theta - \cot\alpha) \quad (6.18)$$

$(M_{Ed}/z) + \Delta F_{td} \leq M_{Ed,max}/z$ , where  $M_{Ed,max}$  is the maximum moment along the beam

### Additional tensile force in the longitudinal reinforcement ... caused by shear

(2) EN Section 9:

→ For members without shear reinforcement

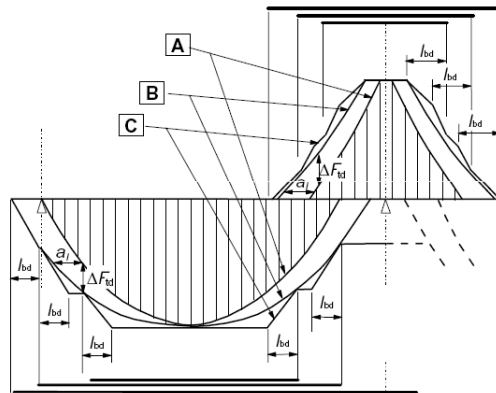
$\Delta F_{td}$  may be estimated by **shifting the moment curve** (in the region cracked in flexure) a distance  $a_l = d$  in the unfavourable direction .

→ For members with shear reinforcement

This "shift rule" may also be used as an alternative to approach (1), where:

$$a_l = z (\cot\theta - \cot\alpha) / 2 \quad (9.2)$$

### Additional tensile force in the longitudinal reinforcement ... caused by shear



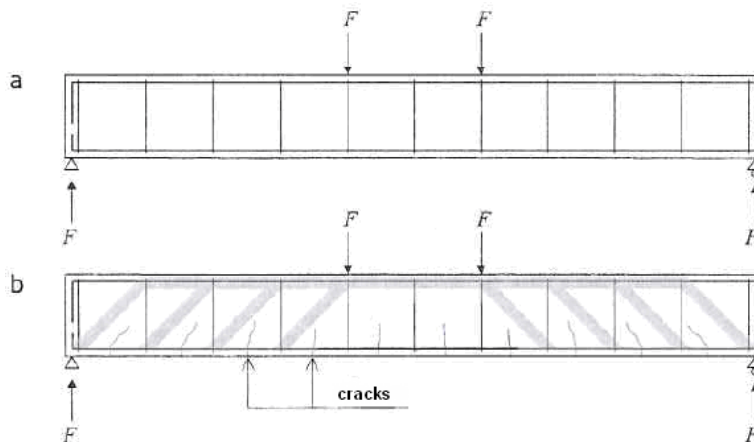
[A] - Envelope of  $M_{Ed}/z + N_{Ed}$  [B] - acting tensile force  $F_s$  [C] - resisting tensile force  $F_{Rs}$

The curtailment of longitudinal reinforcement, taking into account the effect of inclined cracks and the resistance of reinforcement bars within their anchorage lengths.

(As a conservative simplification the contribution of the anchorage may be ignored.)

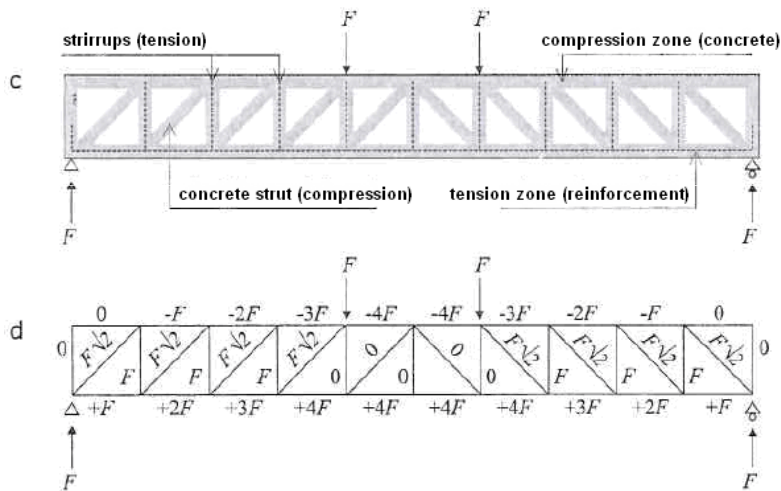
## Section 6 Ultimate limit states (ULS)

### Explanation for the additional tensile force



a) reinforced concrete beam b) cracked beam with compression struts, after loading

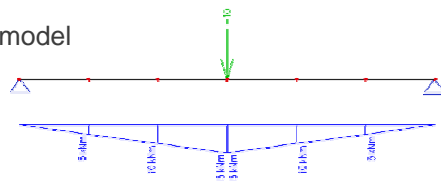
### Explanation for the additional tensile force



c) truss analogy ( $\theta = 45^\circ$ ) d) internal forces in the members of the truss

### Explanation for the additional tensile force

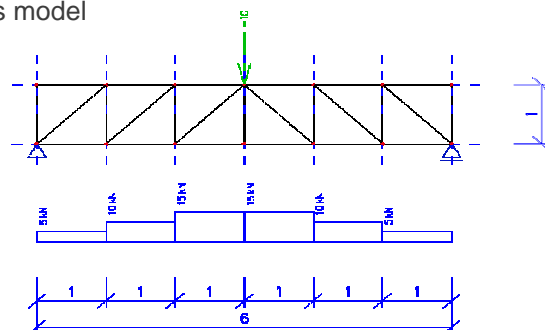
#### ■ Beam model



$$N = M / z$$

assume  $z = 1$

#### ■ Truss model



### Conclusion:

Design of reinforcement according to beam model is unsafe  $\rightarrow$  shift of M-line

### General

The torsional resistance of a section may be calculated on the basis of a **thin-walled closed section**, in which equilibrium is satisfied by a closed shear flow.

- Solid section → equivalent thin-walled section
- Complex shape (e.g. T-sections) → series of equivalent thin-walled section, where the total torsional resistance = sum of the capacities of the individual elements
- Non-solid sections → equivalent wall thickness  $\leq$  actual wall thickness

### Design procedure

#### Definitions

The shear stress in a wall  $i$  of a section subject to a pure torsional moment:

$$\tau_{t,i} t_{ef,i} = T_{Ed} / 2A_k \quad (6.26)$$

The shear force  $V_{Ed,i}$  in a wall  $i$  due to torsion is given by:

$$V_{Ed,i} = \tau_{t,i} t_{ef,i} Z_i \quad (6.27)$$

where

$T_{Ed}$  is the applied design torsion

$\tau_{t,i}$  is the torsional shear stress in wall  $i$

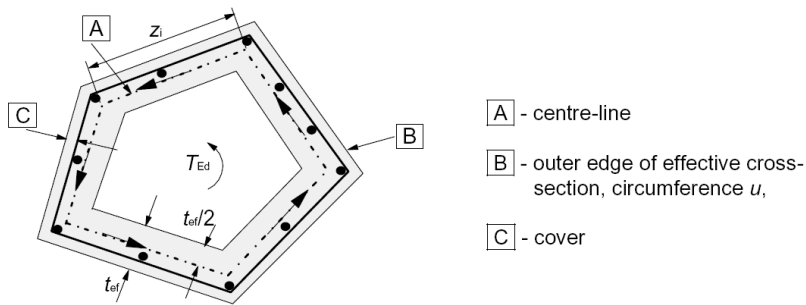
$A_k$  is the area enclosed by the centre-lines of the connecting walls, including inner hollow areas

$t_{ef,i}$  is the effective wall thickness, which may be taken as  $A/u$

$A$  is the total area of the cross-section within the outer circumference, including inner hollow areas

$u$  is the outer circumference of the cross-section

$z_i$  is the side length of wall  $i$



### Longitudinal reinforcement for torsion $\Sigma A_{sl}$

$$\Sigma A_{sl} = (T_{Ed} \cot \theta u_k) / (2 A_k f_{yd}) \quad (6.28)$$

where:

$u_k$  is the perimeter of the area  $A_k$

$f_{yd}$  is the design yield stress of the longitudinal reinforcement  $A_{sl}$

$\theta$  is the angle of compression struts

In compressive chords: The longitudinal reinf. may be reduced in proportion to the available compressive force.

In tensile chords: The longitudinal reinf. for torsion should be added to the other reinforcement. It should be distributed over the length of side,  $z_i$ , but for smaller sections it may be concentrated at the ends of this length.

### Transverse reinforcement for torsion (and shear)

The effects of torsion (T) and shear (S) may be superimposed, assuming the same value for the strut inclination  $\theta$ . Limits for  $\theta$  are given in (6.7N).

This means  $V_{Ed} = V_{Ed}(S) + V_{Ed}(T)$

$$\text{where: } V_{Ed}(T) = \sum V_{Ed,i} \quad (6.26) - (6.27)$$

$$\text{For each wall } i: V_{Ed,i} = \tau_{t,i} t_{ef,i} z_i = (T_{Ed} z_i) / (2 A_k) \quad (6.26) - (6.27)$$

$$\text{In practice: } V_{Ed} = V_{Rd,s} = (A_{sw}/s) z f_{ywd} \cot \theta \quad (6.8)$$

### Specific conditions to be checked: Shear – Torsion interaction diagrams

#### 1<sup>st</sup> Condition

In order not to exceed the bearing capacity of the concrete struts for a member subjected to torsion and shear, the following condition should be satisfied:

$$(T_{Ed} / T_{Rd,max}) + (V_{Ed} / V_{Rd,max}) \leq 1 \quad (6.29)$$

where:

$T_{Ed}$  is the design torsional moment &  $V_{Ed}$  is the design transverse force

$T_{Rd,max}$  is the design torsional resistance moment

$$T_{Rd,max} = 2 v \alpha_{cw} f_{cd} A_k t_{ef,i} \sin \theta \cos \theta \quad (6.30)$$

where  $v$  follows from (6.6N) and  $\alpha_{cw}$  from (6.9)

$V_{Rd,max}$  is the maximum design shear resistance according to (6.9) or (6.14). In solid cross-sections the full width of the web may be used to determine  $V_{Rd,max}$ .

### Specific conditions to be checked: Shear – Torsion interaction diagrams

#### 2<sup>nd</sup> Condition

For approximately rectangular solid sections, only minimum reinforcement is required if the following condition is satisfied:

$$(T_{Ed} / T_{Rd,c}) + (V_{Ed} / V_{Rd,c}) \leq 1 \quad (6.31)$$

where :

$T_{Rd,c}$  is the torsional cracking moment, which may be determined by setting  $\tau_{t,i} = f_{ctd}$

$V_{Rd,c}$  follows from (6.2)

Minimum transverse reinforcement → see  $\rho_{w,min}$  (9.5N)

### Torsion in Scia Engineer

Not taken into account by default !

<input checked="" type="checkbox"/> <b>Calculation</b>	
<input checked="" type="checkbox"/> <b>General</b>	
Number of iteration steps	100
Precision of iteration [%]	1
Limit value for checks [-]	1.00
User defined and end sections only	<input type="checkbox"/> no
Concrete area weakened by reinforcement bars	<input type="checkbox"/> no
Concrete area weakened by prestressed reinforcement	<input type="checkbox"/> no
For design calculations of 1D members, consider longitud...	<input checked="" type="checkbox"/> yes
Check torsion	<input checked="" type="checkbox"/> yes
Check shear of construction joint	<input type="checkbox"/> no
Calculation of additional force caused by shear and torsion	Method according to 9.2.1.3 ▼

! Torsion reinforcement is only calculated for the walls  $i$  // local  $z$  axis of the beam !

For  $A_{sl}$ : all of the required reinf. is distributed over the walls  $i$  // local  $z$  axis

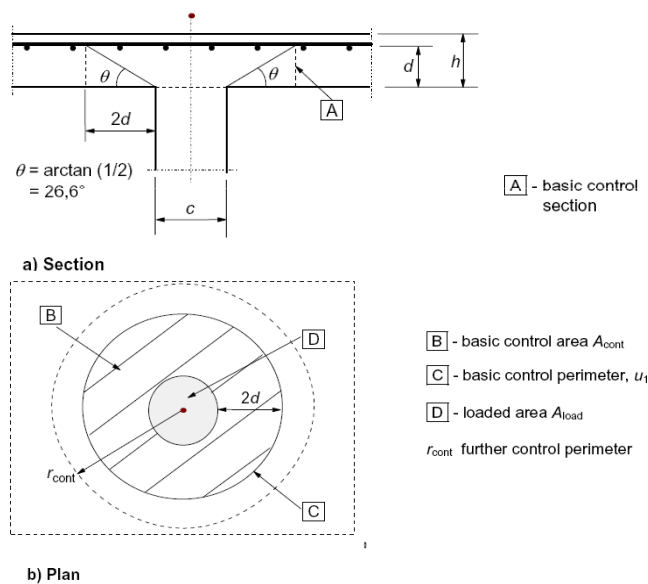
For  $A_{sw}$ : only the required reinf. for  $V_z(T)$  is calculated, not the one for  $V_y(T)$

### General

- Punching = 'extension' of the shear principles
- Punching shear results from a **concentrated load or reaction**, acting on a small area  $A_{\text{load}}$  (the loaded area of a slab or a foundation)
- Verification model for checking punching failure at the ULS, based on control perimeters where checks will be performed.

## Section 6 Ultimate limit states (ULS)

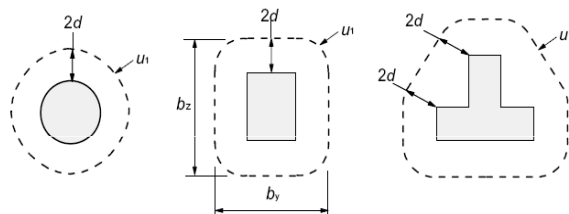
### Verification model for checking punching failure at the ULS:





### Basic control perimeter $u_1$

Normally taken at a distance  $2d$  from the loaded area:



The effective depth  $d_{\text{eff}}$  of the slab is assumed constant:

$$d_{\text{eff}} = (d_y + d_z) / 2 \quad (6.32)$$

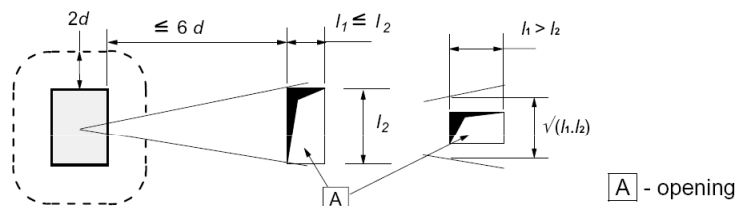
where  $d_y$  and  $d_z$  are the effective depths of the reinf. in 2 orthogonal directions

In case the concentrated force is opposed by a high pressure (e.g. soil pressure on a column base), control perimeters at a distance less than  $2d$  should be considered.

## Section 6 Ultimate limit states (ULS)

### Basic control perimeter $u_1$

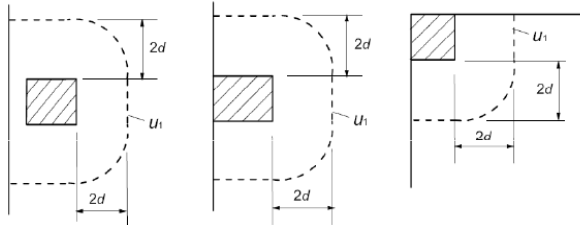
In case of a loaded area near an opening:



If the shortest distance between the perimeter of the loaded area and the edge of the opening  $\leq 6d$ , the part of the control perimeter contained between two tangents is considered ineffective.

### Basic control perimeter $u_1$

In case of a loaded area near an edge or corner:



If the distance to the edge or corner is smaller than  $d$ , special edge reinforcement should always be provided, see EN § 9.3.1.4.

### Further perimeters $u_i$

Further perimeters  $u_i$  should have the same shape as the basic control perimeter  $u_1$ .

## Section 6 Ultimate limit states (ULS)

### Design procedure

Based on checks at the face of the column and at the basic control perimeter  $u_1$ .

If shear reinforcement is required, a further perimeter  $u_{out,ef}$  should be found where shear reinforcement is no longer required.

### Definition of design shear resistances [MPa]

$V_{Rd,c}$  = design value of the punching shear resistance of a slab *without* punching shear reinforcement along the control section considered

$V_{Rd,cs}$  = design value of the punching shear resistance of a slab *with* punching shear reinforcement along the control section considered

$V_{Rd,max}$  = design value of the *maximum* punching shear resistance along the control section considered

### Design procedure

#### Checks to be performed

- Check at the face of the column, or at the perimeter of the loaded area (perimeter  $u_0$ ):

$$v_{Ed0} \leq v_{Rd,max}$$

with  $v_{Ed0}$  the design shear stress at the column perimeter  $u_0$

- Check at the basic control perimeter  $u_1$ :

If  $v_{Ed} \leq v_{Rd,c}$ : Punching shear reinforcement is not required

If  $v_{Ed} > v_{Rd,c}$ : Punching shear reinforcement has to be provided acc. to (6.52)

with  $v_{Ed}$  the design shear stress at the basic control perimeter  $u_1$

## Section 6 Ultimate limit states (ULS)

### Design procedure

**Remark:** Where the support reaction is eccentric with regard to the control perimeter, the maximum shear stress should be taken as:

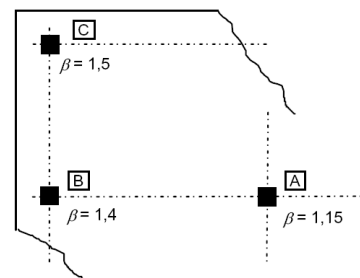
$$v_{Ed} = \beta V_{Ed} / u_i d \quad (6.38)$$

where:

$\beta$  can be calculated with the formulas in EN § 6.4.3(3)-(4)-(5)

Often, approximate values for  $\beta$  may be used:

(recommended values for internal (A), edge (B) and corner(C) columns)



### Design procedure

**Remark:** In case of a foundation slab, the punching shear force  $V_{Ed}$  may be reduced due to the favourable action of the soil pressure.

$$V_{Ed,red} = V_{Ed} - \Delta V_{Ed} \quad (\text{for concentric loading}) \quad (6.48)$$

where:

$V_{Ed}$  is the applied shear force

$\Delta V_{Ed}$  is the net upward force within the control perimeter considered, i.e. upward pressure from soil minus self weight of base

$$v_{Ed} = V_{Ed,red} / u d \quad (6.49)$$

Remember: Consider control perimeters *within*  $2d$  from the periphery of the column.

### $V_{Ed} \leq V_{Rd,c}$ : No punching shear reinforcement required

**Design punching shear resistance of a slab *without* shear reinforcement  $v_{Rd,c}$**

$$v_{Rd,c} = C_{Rd,c} k (100 \rho_l f_{ck})^{1/3} + k_1 \sigma_{cp} \geq (v_{min} + k_1 \sigma_{cp}) \quad (6.47)$$

where:

$$C_{Rd,c} = 0,18 / \gamma_c \quad (\text{recommended value})$$

$$v_{min} = 0,035 k^{3/2} \cdot f_{ck}^{1/2} \quad (\text{recommended value, 6.3N})$$

$$k_1 = 0,1 \quad (\text{recommended value})$$

→ Analogy with (6.2a) and (6.2b)

### $V_{Ed} > V_{Rd,c}$ : Punching shear reinforcement required

Design punching shear resistance of a slab *with* shear reinforcement  $v_{Rd,cs}$

$$v_{Rd,cs} = 0,75 v_{Rd,c} + 1,5 (d/s_r) A_{sw} f_{ywd,ef} (1/(u_1 d)) \sin \alpha \quad (6.52)$$

where:

$A_{sw}$  is the area of one perimeter of shear reinforcement around the column [mm<sup>2</sup>]

$s_r$  is the radial spacing of perimeters of shear reinforcement [mm]

$f_{ywd,ef}$  is the effective design strength of the punching shear reinforcement,  $f_{ywd,ef} = 250 + 0,25 d \leq f_{ywd}$  [MPa]

$d$  is the mean of the effective depths in the orthogonal directions [mm]

$\alpha$  is the angle between the shear reinforcement and the plane of the slab

→ Analogy with (6.13)

### $V_{Ed} > V_{Rd,c}$ : Punching shear reinforcement required

Design punching shear resistance of a slab *with* shear reinforcement  $v_{Rd,cs}$

$$v_{Rd,cs} = 0,75 v_{Rd,c} + 1,5 (d/s_r) A_{sw} f_{ywd,ef} (1/(u_1 d)) \sin \alpha \quad (6.52)$$

Explanation of the formula:

$$v_{Rd,cs} = 0,75 v_{Rd,c} + v_{Rd,s}$$

- The contribution of the steel comes from the shear reinforcement at  $1,5 d$  from the loaded area.

- The contribution of the concrete is 75% of the resistance of a slab without punching shear reinforcement.

### $V_{Ed} > V_{Rd,c}$ : Punching shear reinforcement required

Design maximum punching shear resistance  $v_{Rd,max}$

$$v_{Ed} = \beta V_{Ed} / u_0 d \leq v_{Rd,max} \quad (6.53)$$

$$v_{Rd,max} = 0,5 v f_{cd} \quad (\text{recommended value})$$

where

$u_0$  for an interior column  $u_0 = \text{length of column periphery [mm]}$

for an edge column  $u_0 = c_2 + 3d \leq c_2 + 2c_1$  [mm]

for a corner column  $u_0 = 3d \leq c_1 + c_2$  [mm]

$c_1, c_2$  are the column dimensions

$v$  see (6.6)

$\beta$  see EN § 6.4.3(3)-(4)-(5)-(6)

## Section 6 Ultimate limit states (ULS)

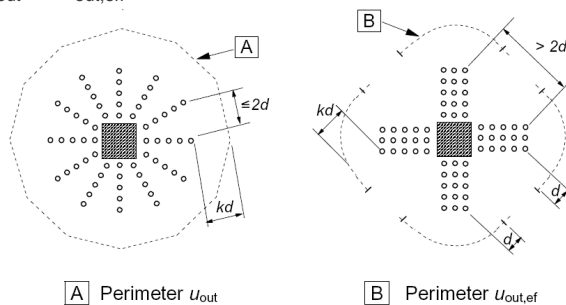
### $V_{Ed} > V_{Rd,c}$ : Punching shear reinforcement required

Control perimeter at which shear reinforcement is no longer required,  $u_{out}$  or  $u_{out,ef}$

$$u_{out,ef} = \beta V_{Ed} / (v_{Rd,c} d) \quad (6.54)$$

The outermost perimeter of shear reinforcement should be placed at a distance  $\leq k d$  within  $u_{out}$  or  $u_{out,ef}$ :

$k = 1,5$  (recommended value)



## Section 7 Serviceability limit states (SLS)

## Section 7 Serviceability limit states (SLS)

### Common serviceability limit states

- Stress limitation
- Crack control
- Deflection control

Other limit states, like vibration, are not covered in this Standard.

### Limitation of the compressive stress in the concrete

under the **characteristic** combination of loads

... to avoid longitudinal cracks, micro-cracks or high levels of creep, where they could result in unacceptable effects on the function of the structure

e. g. To avoid longitudinal cracks, which may lead to a reduction of durability:

Limitation of the compressive stress to a value  $k_1 f_{ck}$ ,

in areas exposed to environments of exposure classes XD, XF and X

where  $k_1 = 0,6$  (recommended value)

Other (equivalent) measures:

- an increase in the cover to reinforcement in the compressive zone
- confinement by transverse reinforcement

### Limitation of the compressive stress in the concrete

under the **quasi-permanent** combination of loads

... to avoid non-linear creep

If  $\sigma_c \leq k_2 f_{ck}$  linear creep may be assumed

If  $\sigma_c > k_2 f_{ck}$  non-linear creep should be considered

where  $k_2 = 0,45$  (recommended value)



### Limitation of the tensile stress in the reinforcement

under the **characteristic** combination of loads

... to avoid inelastic strain, unacceptable cracking or deformation

e.g. To avoid unacceptable cracking or deformation:

Limitation of the tensile stress to a value  $k_3 f_{yk}$

where  $k_3 = 0,8$  (recommended value)

e.g. In case of an imposed deformation:

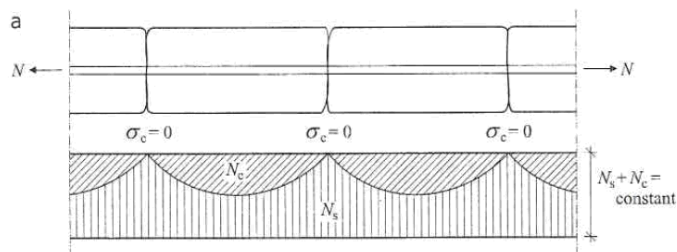
Limitation of the tensile stress to a value  $k_4 f_{yk}$

where  $k_4 = 1$  (recommended value)

## Section 7 Serviceability limit states (SLS)

### Principle

Cracking of an axially loaded reinforced concrete column



**a.**  $N_c$  and  $N_s$  in relation to the pattern of cracks



**b.** Pattern of cracks in case of one large reinforcement bar



**c.** Pattern of cracks in case of four small reinforcement bars

### Limitation of cracking

under the **quasi-permanent** combination of loads

... to guarantee the proper functioning and durability of the structure, and acceptable appearance

- Cracking is normal in reinforced concrete structures!
- Cracks may be permitted to form without any attempt to control their width, provided that they do not impair the functioning of the structure.

### Max. crack width $w_{\max}$

$w_{\max} =$  (recommended values, EN Table 7.1N)

Exposure Class	Reinforced members and prestressed members with unbonded tendons	Prestressed members with bonded tendons
	Quasi-permanent load combination	Frequent load combination
X0, XC1	0,4 <sup>1</sup>	0,2
XC2, XC3, XC4	0,3	0,2 <sup>2</sup>
XD1, XD2, XS1, XS2, XS3		Decompression
<b>Note 1:</b> For X0, XC1 exposure classes, crack width has no influence on durability and this limit is set to guarantee acceptable appearance. In the absence of appearance conditions this limit may be relaxed.		
<b>Note 2:</b> For these exposure classes, in addition, decompression should be checked under the quasi-permanent combination of loads.		

... taking into account the proposed function and nature of the structure and the costs of limiting cracking

### Minimum reinforcement areas

- A minimum amount of bonded reinforcement is required to control cracking in areas where tension is expected.
- The amount may be estimated from equilibrium between the tensile force in concrete just before cracking and the tensile force in reinforcement at yielding.  
(or at a lower stress if necessary to limit the crack width)

## Section 7 Serviceability limit states (SLS)

### Min. reinforcement area $A_{s,min}$ (within the tensile zone)

$$A_{s,min} = (k_c k f_{ct,eff} A_{ct}) / \sigma_s \quad (7.1)$$

where:

$A_{ct}$  is the area of concrete within the tensile zone, just before the formation of the first crack

$\sigma_s$  is the maximum stress permitted in the reinforcement immediately after formation of the crack:  $\sigma_s = f_{yk}$ , unless a lower value is needed to satisfy the crack width limits according to the maximum bar size or spacing (see further)

$f_{ct,eff} = f_{ctm}$  or  $(f_{ctm}(t))$  if cracking is expected earlier than 28 days

$k = 1,0$  for webs with  $h \leq 300$  mm or flanges with widths  $\leq 300$  mm

$= 0,65$  for webs with  $h \leq 800$  mm or flanges with widths  $> 800$  mm

$k_c$  is a coefficient which takes account of the stress distribution within the section immediately prior to cracking and of the change of the lever arm

### Two alternative methods for limitation of cracking:

- Calculation of crack widths,  
to check if  $w_k \leq w_{\max}$
- Control of cracking without direct calculation,  
but by restricting the bar diameter or spacing (simplified method)

### Control of cracking without direct calculation

Where  $A_{s,\min}$  is provided, and for cracks caused mainly by loading, crack widths are unlikely to be excessive if:

**either** the bar diameters (EN Table 7.2N) **or** the bar spacing (EN Table 7.3N) are not exceeded.

- The steel stress should be calculated on the basis of a cracked section under the relevant combination of actions.
- The values in the tables are based on the following assumptions:  $c = 25\text{mm}$ ;  $f_{ct,eff} = 2,9\text{MPa}$ ;  $h_{cr} = 0,5$ ;  $(h-d) = 0,1h$ ;  $k_1 = 0,8$ ;  $k_2 = 0,5$ ;  $k_c = 0,4$ ;  $k = 1,0$ ;  $k_t = 0,4$  and  $k' = 1,0$

### Control of cracking without direct calculation

Steel stress <sup>2</sup> [MPa]	Maximum bar size [mm]		
	$w_k = 0,4$ mm	$w_k = 0,3$ mm	$w_k = 0,2$ mm
160	40	32	25
200	32	25	16
240	20	16	12
280	16	12	8
320	12	10	6
360	10	8	5
400	8	6	4
450	6	5	-

EN Table 7.2N

Maximum bar diameters  
 $\phi_s^*$  for crack control

Steel stress <sup>2</sup> [MPa]	Maximum bar spacing [mm]		
	$w_k = 0,4$ mm	$w_k = 0,3$ mm	$w_k = 0,2$ mm
160	300	300	200
200	300	250	150
240	250	200	100
280	200	150	50
320	150	100	-
360	100	50	-

EN Table 7.3N

Maximum bar spacing  
for crack control

### Calculation of crack widths $w_k$

$$w_k = s_{r,max} (\epsilon_{sm} - \epsilon_{cm}) \quad (7.8)$$

where

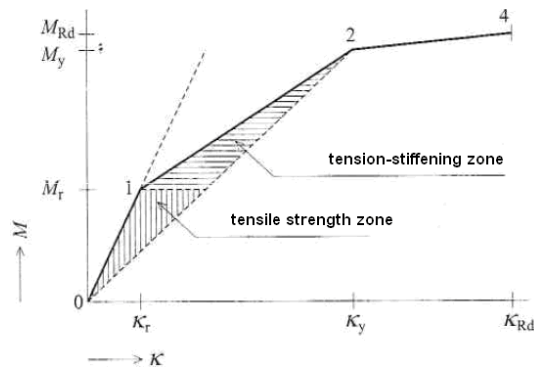
$s_{r,max}$  is the maximum crack spacing

$\epsilon_{sm}$  is the mean strain in the reinforcement under the relevant combination of loads, including the effect of imposed deformations and taking into account the effects of tension stiffening.

$\epsilon_{cm}$  is the mean strain in the concrete between cracks

For the formulas for  $(\epsilon_{sm} - \epsilon_{cm})$  and  $s_{r,max}$ , see EN § 7.3.4

### Principle



$M_r$  = moment of 1<sup>st</sup> cracking

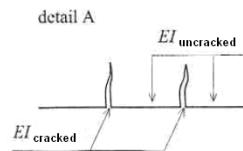
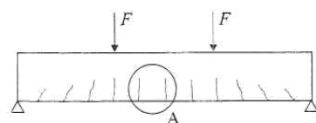
$M_y$  = yielding moment (steel)

$M_{Rd}$  = moment of resistance (failure of concrete under compression)

zone 0-1: no cracks

zone 1-2: cracks arise and widen

zone 2-4: cracks become visibly wide  
(control mechanism to failure)



### Limitation of deflection

under the **quasi-permanent** combination of loads

... to avoid adversely affection of the proper functioning or appearance

Limitation of the calculated sag of a beam, slab or cantilever:

$$1/250 * \text{span}$$

Limitation of deflections that could damage adjacent parts of the structure:

$$1/500 * \text{span} \quad (\text{deflection after construction})$$

### Two alternative methods for limitation of deflection:

- Calculation of deflection,  
to check if the calculated value  $\leq$  the limit value
- Control of deflection without direct calculation,  
but by limiting the span/depth ratio

### Checking deflections by calculation

- Consideration of 2 conditions
  - (I) uncracked condition
  - (II) fully cracked condition

Members which are expected to crack, but may not be fully cracked, will behave in a manner **intermediate** between the uncracked and fully cracked conditions.

### Checking deflections by calculation

- A prediction of behaviour (for members subjected mainly to flexure) is given by:

$$\alpha = \zeta \alpha_{II} + (1 - \zeta) \alpha_I \quad (7.18)$$

where:

$\alpha$  is the deformation parameter considered, e.g. a strain, a curvature, a rotation, or – as a simplification – a deflection

$\alpha_I$ ,  $\alpha_{II}$  are the values for the uncracked and fully cracked conditions

$\zeta$  is a distribution coefficient (allowing for tensioning stiffening at a section)

### Checking deflections by calculation

$$\zeta = 1 - \beta (\sigma_{sr}/\sigma_s)^2 \quad (7.19)$$

$\zeta = 0$  for uncracked sections

$\beta$  is a coefficient taking account of the influence of the duration of the loading

$\beta = 1,0$  for a single short-term loading

$\beta = 0,5$  for sustained loads or many cycles of repeated loading

$\sigma_s$  is the stress in the tension reinforcement calculated on the basis of a cracked section

$\sigma_{sr}$  is the stress in the tension reinforcement calculated on the basis of a cracked section under the loading conditions causing first cracking

**Note:**  $\sigma_{sr}/\sigma_s$  may be replaced by  $M_{cr}/M$  for flexure or  $N_{cr}/N$  for pure tension, where  $M_{cr}$  is the cracking moment and  $N_{cr}$  is the cracking force.



### Checking deflections by calculation

- Taking account of creep

For loads with a duration causing creep, the total deformation including creep may be calculated by using an effective modulus of elasticity for concrete:

$$E_{c,eff} = E_{cm} / [1 + \varphi(\infty, t_0)] \quad (7.20)$$

where:

$\varphi(\infty, t_0)$  is the creep coefficient

### Checking deflections by calculation

- Most rigorous method of assessing deflections:

Compute the **curvatures** at frequent sections along the member

& calculate the deflection by numerical integration

Do this twice,

1<sup>st</sup> time: assuming the whole member to be uncracked (Condition I)

2<sup>nd</sup> time: assuming the member to be fully cracked (Condition II)

then **interpolate** using  $\alpha = \zeta \alpha_{II} + (1 - \zeta) \alpha_I$

### Deflection control in Scia Engineer = Code Dependent Deflection (CDD) calculation

Using  $\alpha = \zeta \alpha_{II} + (1 - \zeta) \alpha_I$  with deformation parameter  $\alpha = \text{reverse stiffness } 1/EI$

Per finite mesh element, an equivalent stiffness  $(EI)_r$  is calculated:

$$(EI)_r = \frac{1}{\frac{\zeta}{(EI)_{II}} + \frac{1-\zeta}{(EI)_I}} \quad (\zeta = 0 \text{ for uncracked sections})$$

$(EI)_I$  : short term stiffness (uncracked condition)

$E = E_{cm}$   $I = \text{based on total concrete css + reinf. area}$

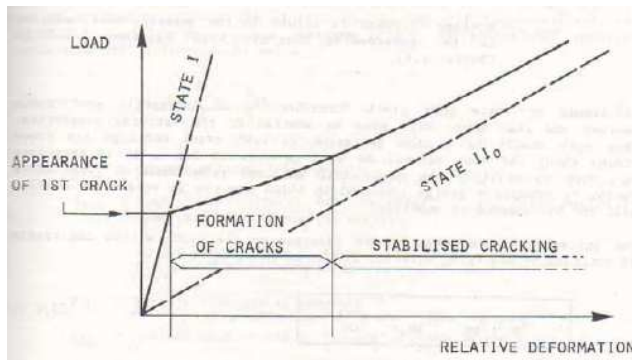
$(EI)_{II}$  : long term stiffness (fully cracked condition)

$E = E_{c,eff}$   $I = \text{based on concrete css under compression + reinf. area}$

## Section 7 Serviceability limit states (SLS)

### Deflection control in Scia Engineer = Code Dependent Deflection (CDD) calculation

The transition from the uncracked state (I) to the cracked state (II) does not occur abruptly, but gradually. From the appearance of the first crack, realistically, a parabolic curve can be followed which approaches the line for the cracked state (II).



→ distribution coefficient  $\zeta$

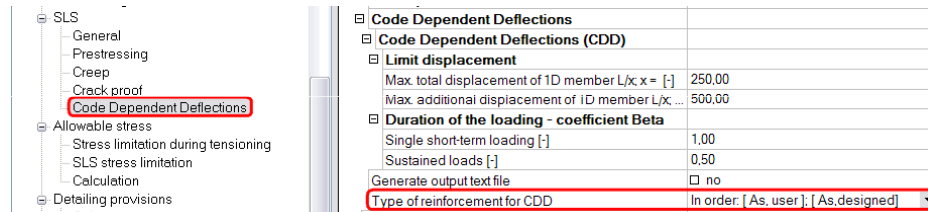
$$(EI)_r = \frac{1}{\frac{\zeta}{(EI)_{II}} + \frac{1-\zeta}{(EI)_I}}$$

## Section 7 Serviceability limit states (SLS)

### Deflection control in Scia Engineer = Code Dependent Deflection (CDD) calculation

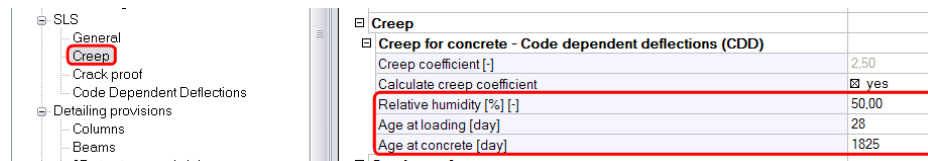
User input in Scia Engineer

- The type of reinforcement for which the CDD calculation will be performed



Code Dependent Deflections (CDD)	
<b>Limit displacement</b>	
Max. total displacement of 1D member $L/x$ , $x = [-]$	250,00
Max. additional displacement of 1D member $L/x$ , ...	500,00
<b>Duration of the loading - coefficient Beta</b>	
Single short-term loading $[-]$	1,00
Sustained loads $[-]$	0,50
Generate output text file	<input type="checkbox"/> no
Type of reinforcement for CDD	In order: [ As, user ]; [ As,designed ]

- The parameters for calculation of the creep coefficient (acc. to EN Annex B1)

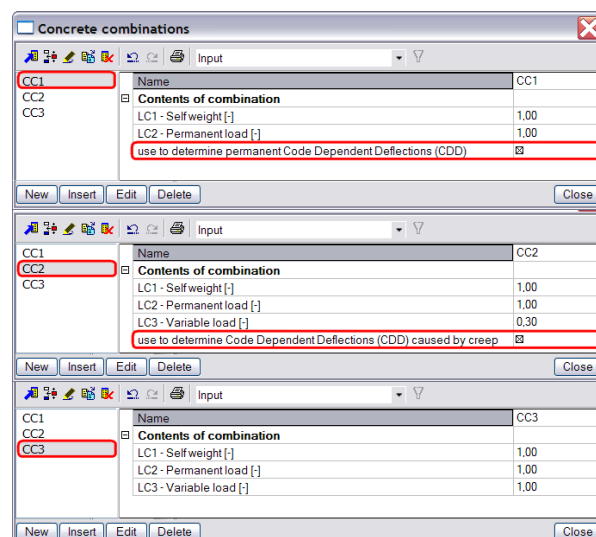


Creep	
<b>Creep for concrete - Code dependent deflections (CDD)</b>	
Creep coefficient $[-]$	2,50
Calculate creep coefficient	<input checked="" type="checkbox"/> yes
Relative humidity [%] $[-]$	50,00
Age at loading [day]	28
Age at concrete [day]	1825

144

## Section 7 Serviceability limit states (SLS)

### Deflection control in Scia Engineer = Code Dependent Deflection (CDD) calculation



Name	Contents of combination
CC1	LC1 - Self weight $[-]$ : 1,00 LC2 - Permanent load $[-]$ : 1,00 use to determine permanent Code Dependent Deflections (CDD) <input checked="" type="checkbox"/>
CC2	LC1 - Self weight $[-]$ : 1,00 LC2 - Permanent load $[-]$ : 1,00 LC3 - Variable load $[-]$ : 0,30 use to determine Code Dependent Deflections (CDD) caused by creep <input checked="" type="checkbox"/>
CC3	LC1 - Self weight $[-]$ : 1,00 LC2 - Permanent load $[-]$ : 1,00 LC3 - Variable load $[-]$ : 1,00

#### Concrete combinations

CC1 (Immediate effect)

1,00 SW + 1,00 PL

CC2 (Creep effect)

1,00 SW + 1,00 PL + 0,30 VL

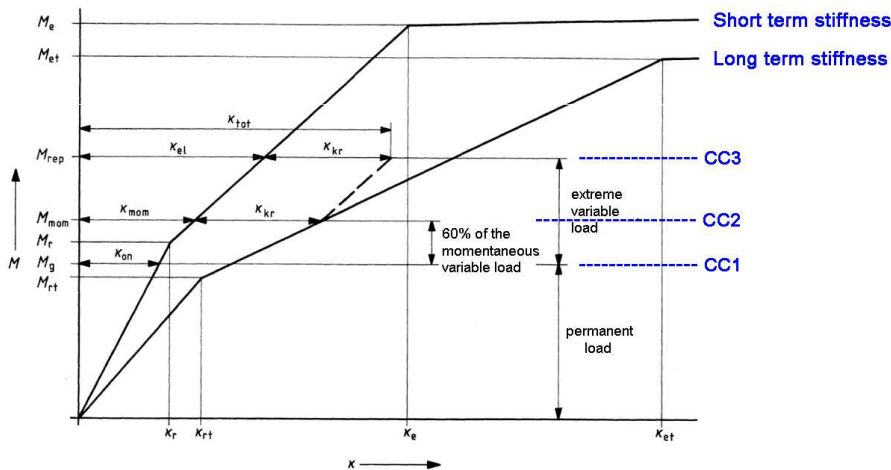
CC3 (Total effect)

1,00 SW + 1,00 PL + 1,00 VL

145

### Deflection control in Scia Engineer = Code Dependent Deflection (CDD) calculation

3 Concrete combinations ~  $M_k$  diagram used by NEN (Dutch code)



146

## Section 7 Serviceability limit states (SLS)

### CDD versus PNL calculation in Scia Engineer

- CDD (Code Dependent Deflection) calculation
  - The formulas take into account the influence of cracks and creep
  - Quasi non-linear calculation: EI is calculated according to approximate formulas
  - Code dependent
  
- PNL (Physical Non Linear) calculation
  - Takes into account the non-linear behaviour of materials, the influence of cracks and creep
  - Real non-linear calculation: EI is calculated iteratively
  - Code independent

147

## Section 8 Detailing of reinforcement - General

148

## Section 8 Detailing of reinforcement - General

### Min. bar spacing $s_{\min}$

The minimum clear distance (horizontal and vertical) between parallel bars should be

$$s_{\min} = \max \{ k_1 \cdot \phi ; (d_g + k_2 \text{ mm}) ; 20 \text{ mm} \}$$

where:

$d_g$  is the maximum aggregate size

$k_1 = 1 \text{ mm}$  (recommended value)

$k_2 = 5 \text{ mm}$  (recommended value)

... such that the concrete can be placed and compacted satisfactorily (by vibrators) for the development of adequate bond

149

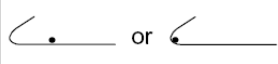
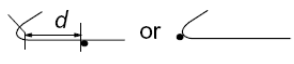
### Min. mandrel diameter $\phi_{m,min}$

$\phi_{m,min} =$  (recommended values, EN Table 8.1N)

#### a) for bars and wire

Bar diameter	Minimum mandrel diameter for bends, hooks and loops (see Figure 8.1)
$\phi \leq 16 \text{ mm}$	$4\phi$
$\phi > 16 \text{ mm}$	$7\phi$

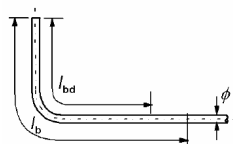
#### b) for welded bent reinforcement and mesh bent after welding

Minimum mandrel diameter	
	
$5\phi$	$d \geq 3\phi : 5\phi$ $d < 3\phi$ or welding within the curved zone: $20\phi$
<b>Note:</b> The mandrel size for welding within the curved zone may be reduced to $5\phi$ where the welding is carried out in accordance with prEN ISO 17660 Annex B	

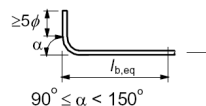
... to avoid bending cracks in the bar, and to avoid failure of the concrete inside the bend of the bar

## Section 8 Detailing of reinforcement - General

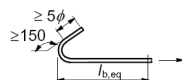
### Methods of anchorage



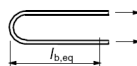
a) Basic tension anchorage length,  $l_b$ , for any shape measured along the centreline



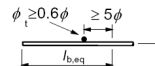
b) Equivalent anchorage length for standard bend



c) Equivalent anchorage length for standard hook



d) Equivalent anchorage length for standard loop



e) Equivalent anchorage length for welded transverse bar

... to ensure that the bond forces are safely transmitted to the concrete, avoiding longitudinal cracking or spalling

### Ultimate bond stress

Design value of ultimate bond stress  $f_{bd}$

$$f_{bd} = 2,25 \eta_1 \eta_2 f_{ctd} \quad (8.2)$$

where:

$f_{ctd}$  is the design value of concrete tensile strength

$\eta_1$  is a coefficient related to the quality of the bond condition and the position of the bar during concreting:

$\eta_1 = 1,0$  ('good' conditions)

$\eta_1 = 0,7$  (all other cases)

$\eta_2$  is related to the bar diameter:

$\eta_2 = 1,0$  for  $\phi \leq 32$  mm

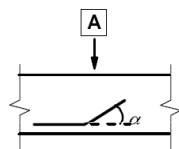
$\eta_2 = (132 - \phi)/100$  for  $\phi > 32$  mm

The ultimate bond strength shall be sufficient to prevent bond failure.

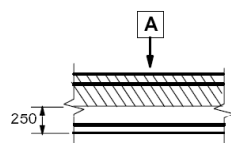
## Section 8 Detailing of reinforcement - General

### Ultimate bond stress

Description of bond conditions

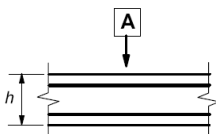


a)  $45^\circ \leq \alpha \leq 90^\circ$

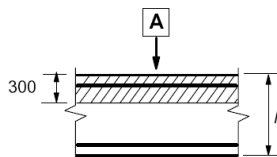


c)  $h > 250$  mm

Direction of concreting



b)  $h \leq 250$  mm



d)  $h > 600$  mm

a) & b) 'good' bond conditions for all bars

c) & d) unhatched zone – 'good' bond conditions  
hatched zone – 'poor' bond conditions

### Basic anchorage length

**Basic required anchorage length  $l_{b,rqd}$**

$$l_{b,rqd} = (\phi / 4) \cdot (\sigma_{sd} / f_{bd}) \quad (8.3)$$

... for anchoring the force  $A_s \cdot \sigma_{sd}$  in a straight bar, assuming constant bond stress  $f_{bd}$  and where  $\sigma_{sd}$  is the design stress of the bar at the position from where the anchorage is measured from

### Design anchorage length

**Min. anchorage length  $l_{b,min}$**

$$l_{b,min} \geq \max \{ 0,3l_{b,rqd} ; 10\phi ; 100 \text{ mm} \} \quad (\text{anchorages in tension}) \quad (8.6)$$

$$l_{b,min} \geq \max \{ 0,6l_{b,rqd} ; 10\phi ; 100 \text{ mm} \} \quad (\text{anchorages in compression}) \quad (8.7)$$

**Design anchorage length  $l_{bd}$**

$$l_{bd} = \alpha_1 \alpha_2 \alpha_3 \alpha_4 \alpha_5 l_{b,rqd} \geq l_{b,min} \quad (8.4)$$

where

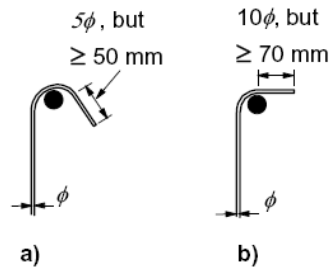
$\alpha_1, \alpha_2, \alpha_3, \alpha_4$  and  $\alpha_5$  are coefficients given in EN Table 8.2,

depending on shape of bar, concrete cover, type of confinement



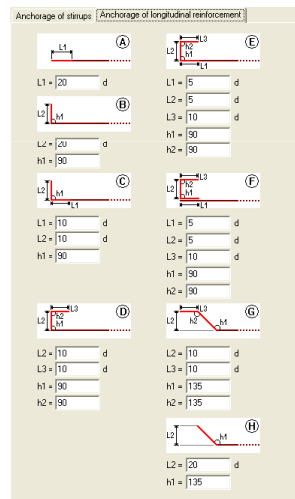
### Methods of anchorage

- by means of bends and hooks, or by welded transverse reinforcement
- a bar should be provided inside each hook or bend

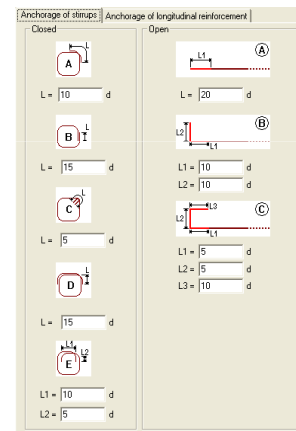


### Anchorage types in Scia Engineer

- Longitudinal reinforcement



### Stirrup reinforcement



### Specifications

- unless stated otherwise, same rules as for individual bars apply
- all the bars in a bundle should have the same characteristics (type and grade) & similar sizes: max. ratio of diameters = 1,7
- in design, the bundle is replaced by a **notional bar** having the same sectional area and the same centre of gravity as the bundle + an equivalent diameter

### Equivalent diameter $\phi_n$

$$\phi_n = \phi \sqrt{n_b} \leq 55 \text{ mm} \quad (8.14)$$

where

$n_b$  is the number of bars in the bundle,

$n_b \leq 4$  (for vertical bars in compression & bars in a lapped joint)

$n_b \leq 3$  (for all other cases)

## Section 9 Detailing of members & particular rules

### Longitudinal reinforcement

#### Min. reinforcement area $A_{s,min}$

$$A_{s,min} = 0,26 (f_{ctm}/f_{yk}) b_t d \geq 0,0013 b_t d \quad (\text{recommended value, 9.1N})$$

where:

$b_t$  is the mean width of the tension zone

$f_{ctm}$  according to EN Table 3.1

See also EN Section 7 for  $A_{s,min}$  to control cracking.

#### Max. reinforcement area $A_{s,max}$ (outside lap locations)

$$A_{s,max} = 0,04 A_c \quad (\text{recommended value})$$

→ Min. areas in order to prevent a brittle failure in the reinforcement steel;  
Max. areas to prevent sudden failure of the concrete compression zone

### Shear reinforcement

#### Stirrup angle $\alpha$

$\alpha$  = between  $45^\circ$  and  $90^\circ$  to the longitudinal axis of the structural element

#### Min. shear reinforcement ratio

$$\rho_w = A_{sw} / (s b_w \sin \alpha) \geq \rho_{w,min} \quad (9.4)$$

where:

$A_{sw}$  is the area of shear reinforcement within length  $s$

$s$  is the spacing of the shear reinforcement along the longitudinal axis of the member

$b_w$  is the breadth of the web of the member

$$\rho_{w,min} = (0,08 \sqrt{f_{ck}}) / f_{yk} \quad (\text{recommended value, 9.5N})$$

### Shear reinforcement

Max. longitudinal spacing  $s_{l,max}$

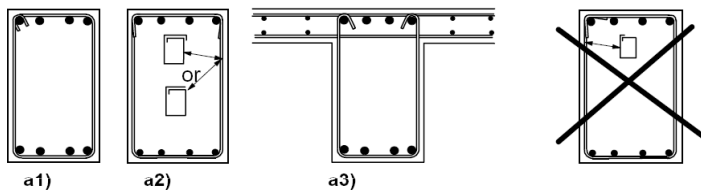
$$s_{l,max} = 0,75 d (1 + \cot \alpha) \quad (\text{recommended value, } 9.6N)$$

Max. transverse spacing of the legs  $s_{t,max}$

$$s_{t,max} = 0,75 d \leq 600 \text{ mm} \quad (\text{recommended value, } 9.8N)$$

## Section 9 Detailing of members & particular rules

### Torsion reinforcement



a) recommended shapes

b) not recommended shape

Longitudinal spacing of the **torsion links** should not exceed

$$u / 8$$

$$s_{l,max} = 0,75 d (1 + \cot \alpha) \quad (\text{recommended value, } 9.6N)$$

the lesser dimension of the beam cross-section

The **longitudinal bars** have to be distributed uniformly, with max. spacing of 350 mm.

### Flexural reinforcement

#### Min. & max. reinforcement areas $A_{s,min}$ & $A_{s,max}$

Same requirements as for beams (see EN § 9.2)

#### Max. spacing $s_{max,slabs}$

$s_{max,slabs} =$  (recommended values)

- for the principal reinforcement:  $3h \leq 400 \text{ mm}$
- for the secondary reinforcement:  $3,5h \leq 450 \text{ mm}$

where  $h$  is the total depth of the slab

In areas with concentrated loads or areas of maximum moment:

- for the principal reinforcement:  $2h \leq 250 \text{ mm}$
- for the secondary reinforcement:  $3h \leq 400 \text{ mm}$

### Flexural reinforcement

#### Curtailment of longitudinal tension reinforcement

Same requirements as for beams (EN § 9.2):

- calculate additional tensile force,  $\Delta F_{td}$ , or
- estimate  $\Delta F_{td}$  by shifting the moment curve a distance  $a_l = d$

#### One way slabs

Minimum secondary transverse reinforcement = 20% of the principal reinforcement

### Shear reinforcement

#### Min. slab thickness

Minimum depth for a slab in which shear reinforcement is provided = 200 mm

#### Min. shear reinforcement ratio

Same requirements as for beams (see EN § 9.2)

#### Max. longitudinal spacing

$$s_{l,max} = 0,75 d (1 + \cot \alpha) \quad (9.9)$$

#### Max. transverse spacing

$$s_{t,max} = 1,5 d$$

### Punching shear reinforcement

Where punching shear reinforcement is required:

#### Min. area of a link leg (or equivalent) $A_{sw,min}$

$$A_{sw,min} = [0,08 \sqrt{f_{ck}} \cdot (s_r \cdot s_t)] / [f_{yk} \cdot (1,5 \sin \alpha + \cos \alpha)] \quad (9.11)$$

where :

$\alpha$  is the angle between the shear reinforcement and the main steel

$s_r$  is the spacing of shear links in the radial direction

$s_t$  is the spacing of shear links in the tangential direction

#### Min. number of perimeters of link legs

$$= 2$$

### Punching shear reinforcement

Distance between the face of the support and the first link leg perimeter

$$\geq 0,3 d \text{ and } \leq 0,5 d$$

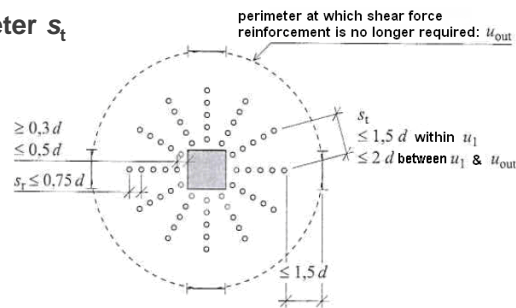
Max. radial spacing of the link leg perimeters  $s_r$

$$s_r \leq 0,75 d$$

Max. tangential spacing around a perimeter  $s_t$

$$s_t \leq 1,5 d \text{ (within } u_1)$$

$$s_t \leq 2 d \text{ (between } u_1 \text{ \& } u_{out})$$



### Longitudinal reinforcement

Min. number of bars

At least one bar at each corner / Min. 4 bars for circular cross-sections

Min. diameter  $\phi_{l,min}$

$$\phi_{l,min} = 8 \text{ mm}$$

(recommended value)

Min. reinforcement area  $A_{s,min}$

$$A_{s,min} = \max \{ 0,10 N_{Ed} / f_{yd} ; 0,002 A_c \}$$

(recommended value, 9.12N)

where:

$N_{Ed}$  is the design axial compression force

Max. reinforcement area  $A_{s,max}$

$$A_{s,max} = 0,04 A_c \text{ (outside lap locations)}$$

(recommended value)

$$A_{s,max} = 0,08 A_c \text{ (at lap locations)}$$

### Transverse reinforcement

#### Min. diameter $\phi_{t,min}$

$$\phi_{t,min} = \max \{ 6 \text{ mm} ; 0,25 \phi_{l,max} \text{ applied} \}$$

#### Max. spacing along the column $s_{cl,tmax}$

$$s_{cl,tmax} \quad (\text{outside lapped joints}) \quad (\text{recommended value})$$
$$= \min \{ 20 \phi_{l,min} \text{ applied} ; \text{the lesser column dimension} ; 400 \text{ mm} \}$$

$$s_{cl,tmax} \quad (\text{near lapped joints if } \phi_{l,max} \text{ applied} > 14\text{mm} \\ \text{\& in the vicinity of a beam or slab})$$
$$= 0,6 \cdot \min \{ 20 \phi_{l,min} \text{ applied} ; \text{the lesser column dimension} ; 400 \text{ mm} \}$$

- The reinforcement design for walls may be derived from a strut-and-tie model.
- For walls subjected predominantly to out-of-plane bending the rules for slabs apply.

### Vertical reinforcement

#### Min. reinforcement area $A_{s,vmin}$

$$A_{s,vmin} = 0,002 A_c \quad (\text{recommended value})$$

#### Max. reinforcement area $A_{s,vmax}$

$$A_{s,vmax} = 0,04 A_c \quad (\text{outside lap locations}) \quad (\text{recommended value})$$
$$A_{s,vmax} = 0,08 A_c \quad (\text{at lap locations})$$

#### Max. bar spacing $s_{vmax}$

$$s_{vmax} = \min \{ 3 \cdot \text{wall thickness} ; 400 \text{ mm} \}$$



### Horizontal reinforcement

To be provided at each surface

**Min. reinforcement area  $A_{s,hmin}$**

$$A_{s,hmin} = \max \{ 0,25 A_{s,v \text{ applied}} ; 0,001 A_c \} \quad (\text{recommended value})$$

**Max. bar spacing  $s_{hmax}$**

$$s_{hmax} = 400 \text{ mm}$$

### Transverse reinforcement

If  $A_{s,v \text{ applied}}$  (total area in the two faces)  $> 0,02 A_c$

then transverse reinforcement should be provided in accordance with the requirements for columns (see EN § 9.5).

## References

- Technical Committee CEN/TC250, *Eurocode 2: Design of concrete structures - Part 1-1: General rules and rules for buildings*, (2004).
- Technical Committee CEN/TC250, *Corrigendum to Eurocode 2: Design of concrete structures - Part 1-1: General rules and rules for buildings*, (2008).
- Braam, C.R., Lagendijk, P., *Constructie-leer Gewapend Beton*, (2008), Cement&BetonCentrum 's-Hertogenbosch.
- Workshop Concrete acc. to EC-EN – Advanced training Scia Engineer.